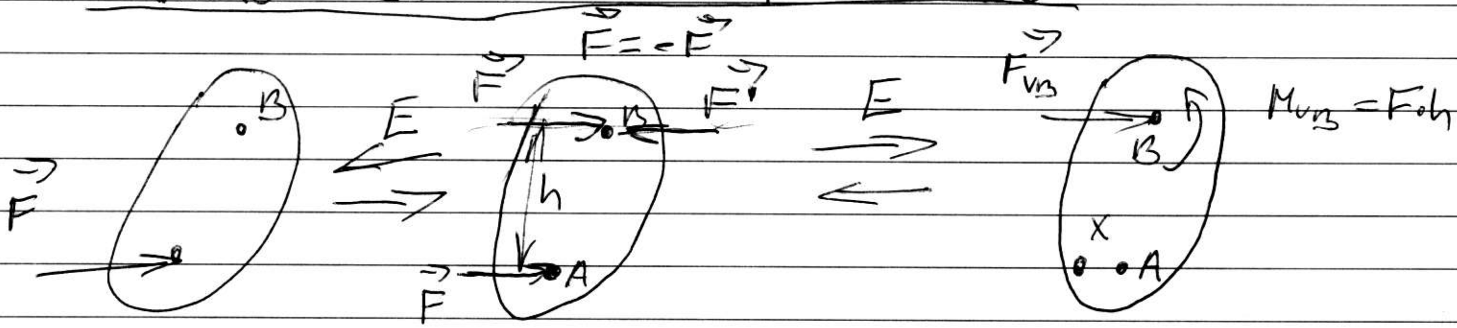
NÁHRADA SILOVÉ SOUSTAVY 1 SILOU



• Silové působení můžeme nahradit v libovolném bodě silovou a momentovou výslednicí. Nyní hledáme takový bod X, ve kterém bude $M_{vX} = 0$. Množina všech těchto bodů (nositelka silové výslednice) se nazývá
OSA SILOVÉ SOUSTAVY!

$$\vec{M}_{vX} = -\vec{BX} \times \vec{F}_v + \vec{M}_{vB} = \vec{0} \Rightarrow \vec{M}_{vB} = \vec{BX} \times \vec{F}$$

• Nutnou (a postačující) podmínkou pro existenci osy silové soustavy je:

$$\boxed{\begin{array}{c} \vec{F}_v \neq \vec{0} \\ \hline I = 0 \end{array}}$$ ($\vec{M}_{vB}$ musí být $\perp$ k $\vec{F}_v$)

jednotkový vektor osy má stejný $\vec{e}$ jako $\vec{F}_v$!

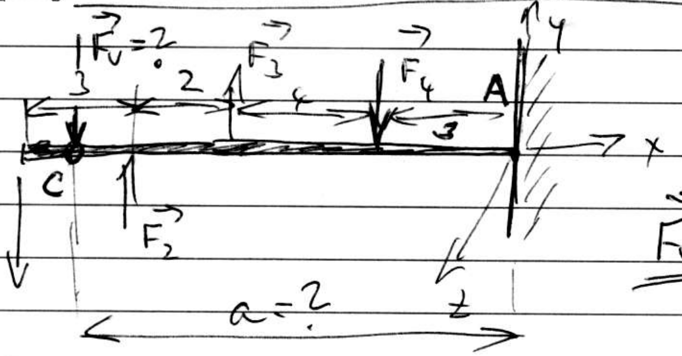
invariant: $I = \vec{M}_{vB} \cdot \vec{F}_v$
(nezávisí na volbě s.s.)

$I = 0$ ... pro rovinnou úlohu splněno vždy, protože $\vec{M}_{vB}\,(0;0;k) \cdot \vec{F}_v\,(i;j;0)$

• Výpočet: $\underbrace{\vec{M}_{vB}}_{\text{známe}} = \underbrace{\vec{BX}}_{\text{neznáme}} \times \underbrace{\vec{F}_v}_{\text{známe}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ X_x - X_B & Y_x - Y_B & Z_x - Z_B \\ F_{vx} & F_{vy} & F_{vz} \end{vmatrix} \overset{B=0}{=} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ X_x & Y_x & Z_x \\ F_{vx} & F_{vy} & F_{vz} \end{vmatrix} \Rightarrow$

$$\Rightarrow \left.\begin{array}{l} M_{Bx} = F_{vz} \cdot Y_x - F_{vy} \cdot Z_x \\ M_{By} = -F_{vz} \cdot X_x + F_{vx} \cdot Z_x \\ M_{Bz} = F_{vy} \cdot X_x - F_{vx} \cdot Y_x \end{array}\right\} \Rightarrow \text{rovnice přímky} = \text{OSA SILOVÉ SOUSTAVY}$$

Př. 1 Nahraďte ve smyslu stat. ekvivalence danou sil. soustavu jedinou silou.



$F_1 = 6N \quad F_3 = 3N$

$F_2 = 5N \quad F_4 = 3{,}5N = 0N$

$$\vec{F_V} = \sum_{i=1}^{4} \vec{F_i} = -6\vec{j} + 5\vec{j} + 3\vec{j} - \overset{0}{3{,}5}\vec{j} = \underline{\underline{-1{,}5\vec{j}\,N}} \quad +2\vec{j}\,N$$

$$\underline{\underline{\vec{M_{VA}}}} = F_1 \cdot 12\vec{k} - F_2 \cdot 9\vec{k} - F_3 \cdot 7\vec{k} + F_4 \cdot 3\vec{k} = \underline{\underline{\overset{6\vec{k}}{16{,}5\vec{k}}\ N\cdot m}}$$

$$\left.\begin{array}{l} \vec{F_V} \neq \vec{0} \\ I = \vec{F_V} \cdot \vec{M_{VA}} = -1{,}5\vec{j} \cdot 16{,}5\vec{k} = 0 \end{array}\right\}$$ lze nahradit jednou silou

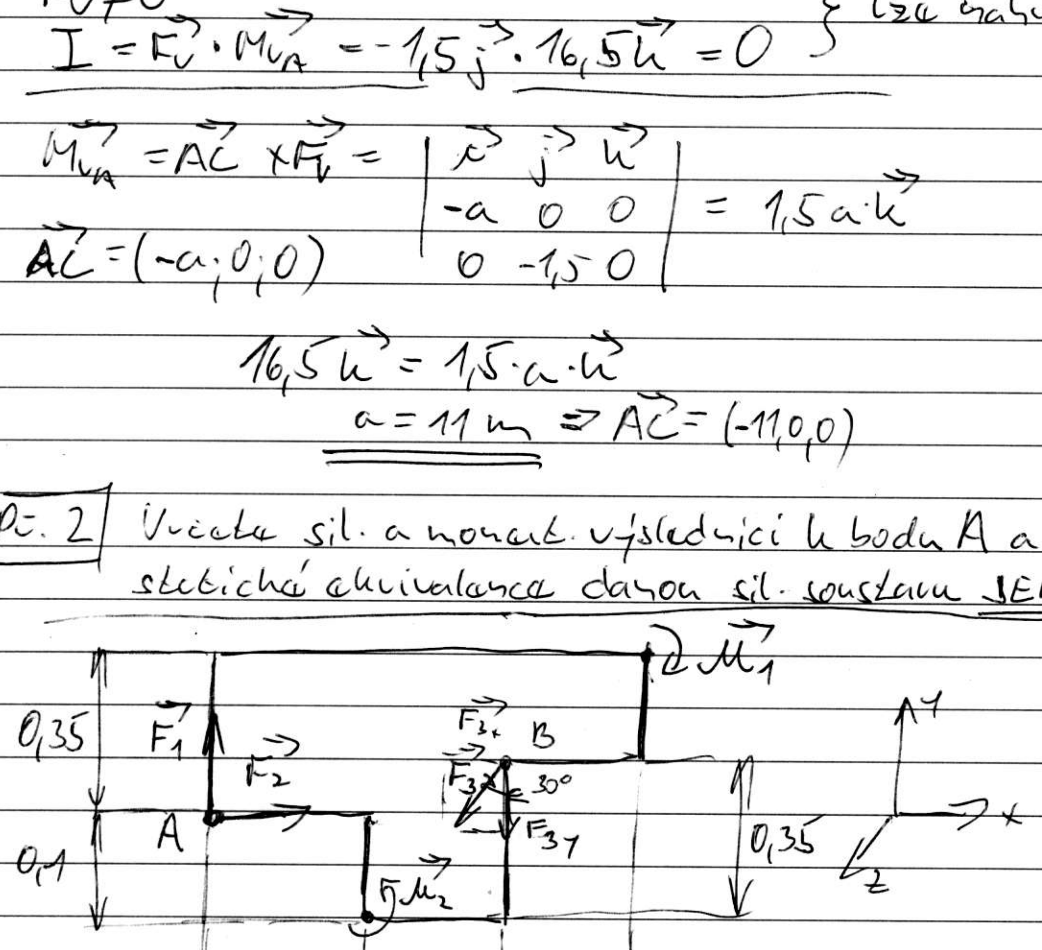
$$\vec{M_{VA}} = \vec{AC} \times \vec{F_V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a & 0 & 0 \\ 0 & -1{,}5 & 0 \end{vmatrix} = 1{,}5a\cdot\vec{k} \qquad \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a & 0 & 0 \\ 0 & 2 & 0 \end{vmatrix} = -2\cdot a\cdot\vec{k}$$

$\vec{AC} = (-a; 0; 0)$

$$16{,}5\vec{k} = 1{,}5\cdot a\cdot\vec{k} \qquad 6\vec{k} = -2\cdot a\cdot\vec{k}$$

$\underline{\underline{a = 11\,m}} \Rightarrow \vec{AC} = (-11, 0, 0) \qquad \underline{\underline{a = 3\,m}} \quad \vec{AC} = (-3, 0, 0)$

Př. 2 Určete sil. a moment. výslednici k bodu A a nahraďte ve smyslu statické ekvivalence danou sil. soustavu JEDINOU SILOU.



$F_1 = 34{,}64N$

$F_2 = 20N$

$F_3 = 40N$

$M_1 = 25\,Nm$

$M_2 = 31\,Nm$

$F_{3x} = F_3 \cdot \sin 30° = 20N$

$F_{3y} = -F_3 \cdot \cos 30° = 34{,}64N$

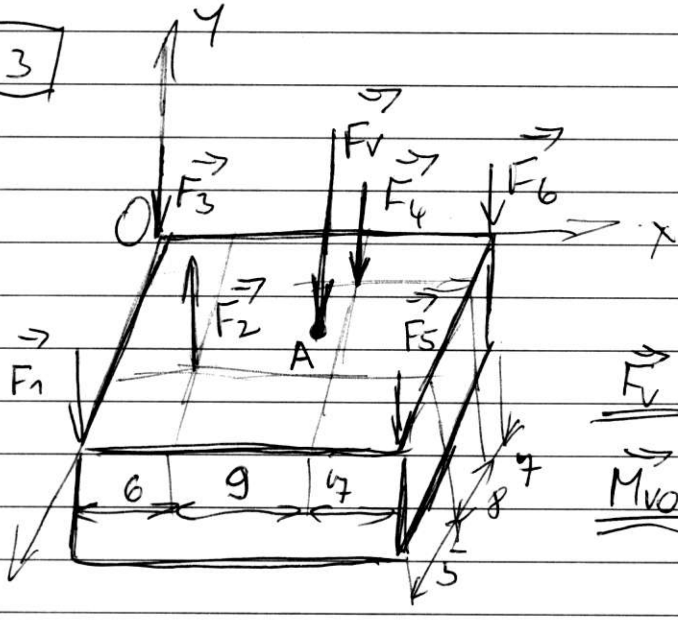
$$\vec{F_V} = F_1\vec{i} + F_2\vec{j} - F_{3y}\vec{j} - F_{3x}\vec{i}$$
$$= (34{,}64 - 34{,}64)\vec{j} + (20 - 20)\vec{i} = \underline{\underline{\vec{0}}}$$

$$\vec{M_{VA}} = \vec{\mathcal{M}_1} + \vec{\mathcal{M}_2} + \vec{AB} \times \vec{F_3} = \vec{\mathcal{M}_1} + \vec{\mathcal{M}_2} + 0{,}25 \cdot F_{3x} \cdot \vec{k} - 0{,}4 \cdot F_{3y} \cdot \vec{k}$$

$$\vec{M_{VA}} = -25\vec{k} + 31\vec{k} - 34{,}64 \cdot 0{,}4 \cdot \vec{k} + 20 \cdot 0{,}25\vec{k} = \underline{\underline{-2{,}86\vec{k}\ N \cdot m = \vec{\mathcal{M}}}}$$

$$\left.\begin{array}{l} \vec{F_V} = \vec{0} \\ \vec{M} \neq \vec{0} \end{array}\right\} \Rightarrow$$ Nelze nahradit 1 silou, lze nahradit sil. dvojicí!

Př. 3



Nahradíte silové působení jediným reprezentantem.

$F_1 = 6kN$; $F_2 = 8kN$; $F_3 = 5kN$
$F_4 = 4kN$; $F_5 = 7kN$; $F_6 = 9kN$

$$\underline{\underline{\vec{F_V}}} = \sum_{i=1}^{6} \vec{F_i} = (-6+8-5-4-7-9)\vec{j} = \underline{\underline{-23\vec{j}\ kN}}$$

$$\underline{\underline{\vec{M_{VO}}}} = F_1 \cdot 20 \cdot \vec{i} - F_2 \cdot 15\vec{i} + F_2 \cdot 6\vec{k} + F_4 \cdot 7 \cdot \vec{i} -$$
$$- F_4 \cdot 15 \cdot \vec{k} + F_5 \cdot 20 \cdot \vec{i} - F_5 \cdot 22 \cdot \vec{k} - F_6 \cdot 22\vec{k} =$$
$$= \underline{\underline{(168\vec{i} - 364\vec{k})\ kN \cdot m}}$$

1) $\vec{F_V} \neq \vec{0}$

1,2

2) $I = \vec{F_V} \cdot \vec{M_V} = (0; -23; 0) \cdot (168; 0; -364) = 0$ → lze nahradit 1 silou!

$$\vec{M_V} = \vec{OA} \times \vec{F_V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_A & y_A & z_A \\ F_{Vx} & F_{Vy} & F_{Vz} \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_A & y_A & z_A \\ 0 & -23 & 0 \end{vmatrix} = \underline{23 z_A \cdot \vec{i} - 23 x_A \cdot \vec{k}}$$

$$168\vec{i} - 364\vec{k} = 23 \cdot z_A \cdot \vec{i} - 23 \cdot x_A \cdot \vec{k} \Rightarrow$$

$$\Rightarrow 168\vec{i} = 23 \cdot z_A \cdot \vec{i} \qquad -364\vec{k} = -23 x_A \cdot \vec{k}$$

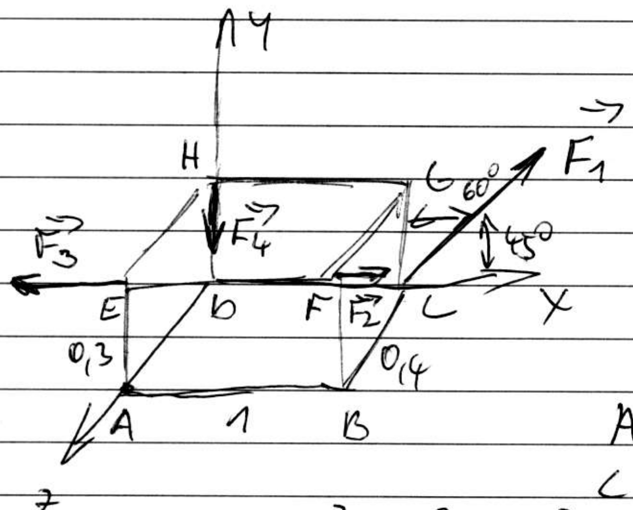
$$\underline{\underline{z_A}} = \frac{168}{23} = \underline{\underline{7{,}3m}} \qquad \underline{\underline{x_A}} = \frac{364}{23} = \underline{\underline{15{,}8m}}$$

Př. 4 Na těleso působí soust. sil.

a) Nahraďte ve smyslu stat. ekvivalence silovou a momentovou výslednicí v b. A

b) Určete momenty zadané sil. soustavy k osám tvořenými hranami kvádru

c) Nahraďte 1 silou



$F_1 = 1200N$

$F_2 = 1250N$

$F_3 = 750N$

$F_4 = 800N$

$0 \leq \gamma \leq \frac{\pi}{2}$

$A = [0;0;0{,}4]$

$C = [1;0;0]$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \Rightarrow \cos\gamma = \frac{1}{2} \Rightarrow \gamma = 60^\circ$

a) $\vec{F_1} = F_1 \cdot \cos 45^\circ \cdot \vec{i} + F_1 \cdot \cos 60^\circ \vec{j} + F_1 \cdot \cos\gamma \cdot \vec{k} =$

$= \sqrt{2} \cdot 600 \cdot \vec{i} + 600\vec{j} + 600\vec{k}$

$\vec{F_v} = F_{1x} \cdot \vec{i} + F_2 \cdot \vec{i} - F_3 \cdot \vec{i} - F_4 \cdot \vec{j} + F_{1y}\vec{j} + F_{1z} \cdot \vec{k} = 1350\vec{i} - 200\vec{j} + 600\vec{k}$

$M_{vA} = \vec{AC} \times \vec{F_1} + \underbrace{\vec{AF} \times \vec{F_2}}_{= \vec{AE} \times \vec{F_2}} + \vec{AE} \times \vec{F_3} + \underbrace{\vec{AH} \times \vec{F_4}}_{= \vec{AD} \times \vec{F_4}}$

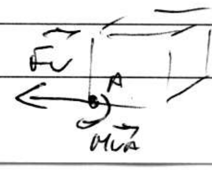
$\vec{M_{F_1,A}} = \vec{AC} \times \vec{F_1} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -0{,}4 \\ \sqrt{2} \cdot 600 & 600 & 600 \end{vmatrix} = -0{,}4 \cdot \sqrt{2} \cdot 600\vec{j} + 600\vec{k} + 0{,}4 \cdot 600\vec{i} - 600\vec{j} =$

$= -939\vec{j} + 240\vec{i} + 600\vec{k}$

$\vec{M_{F_2,A}} = -0{,}3 \cdot 1250 \cdot \vec{k} = -375\vec{k}$

$\vec{M_{F_3,A}} = 0{,}3 \cdot 750\vec{k} = 225\vec{k}$ $\quad$ $\vec{M_{F_4,A}} = -0{,}4 \cdot 800\vec{i} = -320\vec{i}$

$\vec{M_{vA}} = -80\vec{i} - 939\vec{j} + 450\vec{k}$



b) např. $\underline{\underline{\vec{M}_{EH}}} = F_{1x} \cdot 0{,}3 \cdot \vec{u} + F_{1y} \cdot 1 \cdot \vec{u} = \underline{\underline{1449\vec{u}}}$

$\uparrow$

osa EH

$\underline{\underline{\vec{M}_{EA}}} = -F_{1x} \cdot 0{,}4 \cdot \vec{j} - F_{1z} \cdot 1 \cdot \vec{j} = \underline{\underline{-939\vec{j}}}$

$\underline{\underline{\vec{M}_{AB}}} = -F_4 \cdot 0{,}4 \cdot \vec{i} + F_{1y} \cdot 0{,}4 \cdot \vec{i} = \underline{\underline{-80\vec{i}}}$

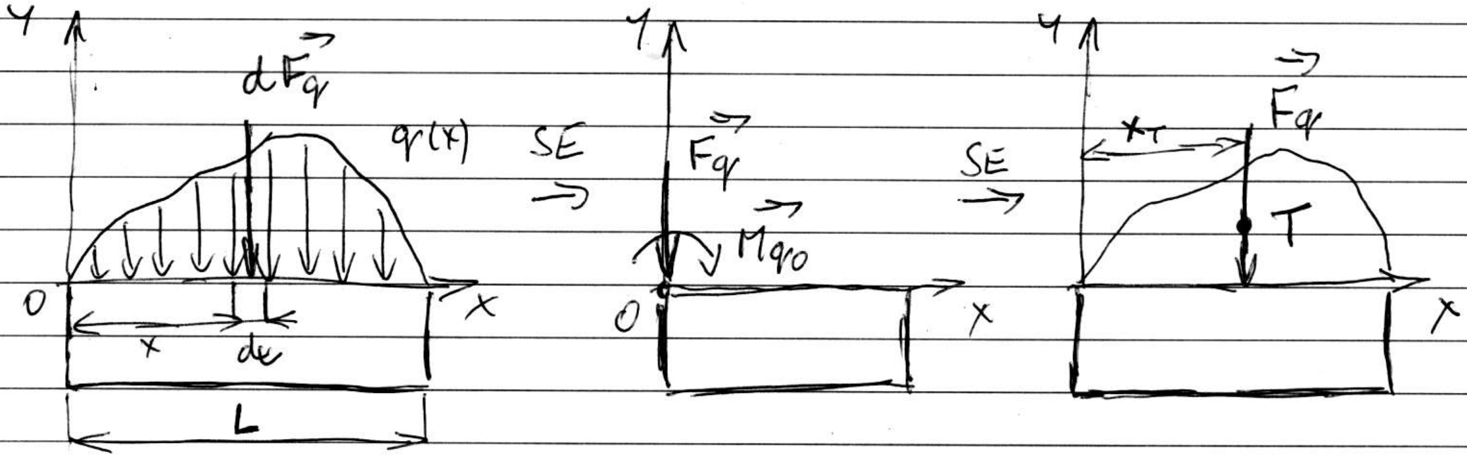
c) $\vec{F_v} \neq \vec{0}$

$I = \vec{M_v} \cdot \vec{F_v} = (-80; -939; 450) \cdot (1350; -200; 600) = 349\,800 \neq 0$

$\Downarrow$

<u>Nelze nahradit jedinou silou</u>

Výslednice rozložení silového působení



$d\vec{F_q} = q(x)\cdot dx$

$$\underline{\underline{F_q = \int_0^L q(x)\,dx}}$$

$d\vec{M_{q0}} = x\cdot dF$

$dM_{q0} = x\cdot q(x)\,dx$

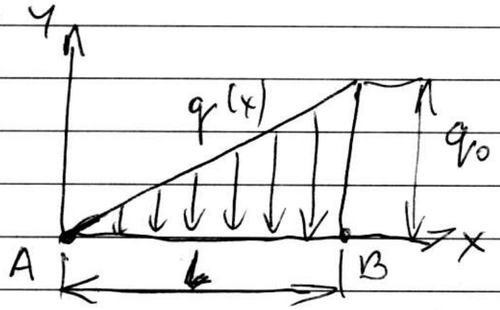
$$M_{q0} = \int_0^L q(x)\cdot x\cdot dx$$

$M_{q0} = F_q\cdot x_T$

$$\underline{\underline{x_T = \frac{\int_0^L q(x)\,x\,dx}{\int_0^L q(x)\,dx}}}$$

Velikost výsl. síly se rovná ploše obrazce a prochází jeho těžištěm

$\boxed{\text{Př. 1}}$



$L = 20\,cm$; $q_0 = 5\,N/cm$

$q(x) = a_0 + a_1\cdot x$

$x = 0:\ q(0) = a_0 = 0$

$x = L:\ q(L) = a_1\cdot L = q_0 \Rightarrow a_1 = \frac{q_0}{L}$

$\underline{q(x)} = \frac{q_0}{L}\cdot x = \frac{5}{20}\cdot x = \underline{\underline{\frac{1}{4}x}}\ [N/cm]$

$\underline{F_q} = \int_0^L dF = \int_0^L q(x)\,dx = \int_0^L \frac{1}{4}x\,dx = \frac{1}{4}\left[\frac{x^2}{2}\right]_0^L = \frac{L^2}{8} = \frac{20^2}{8} = \underline{\underline{50\,N}}$

$\vec{F_q} = -50\,\vec{j}\ [N]$

$\underline{M_{qA}} = \int_0^L x\,dF = \int_0^L q(x)\cdot x\,dx = \int_0^L \frac{1}{4}\cdot x\cdot x\,dx = \frac{1}{4}\left[\frac{x^3}{3}\right]_0^L = \frac{L^3}{12} = \frac{20^3}{12} = \underline{\underline{\frac{2000}{3}\,N\cdot m}}$

$\underline{\underline{\vec{M_{qA}} = -\frac{2000}{3}\,\vec{k}\ [N\cdot m]}}$

$\underline{M_{qA}} = F_q\cdot x_T \Rightarrow x_T = \frac{\frac{2000}{3}}{50} = \underline{\underline{\frac{40}{3}\,cm}} = \underline{\underline{13{,}3\,cm}} = \underline{\underline{\frac{2}{3}L}}$