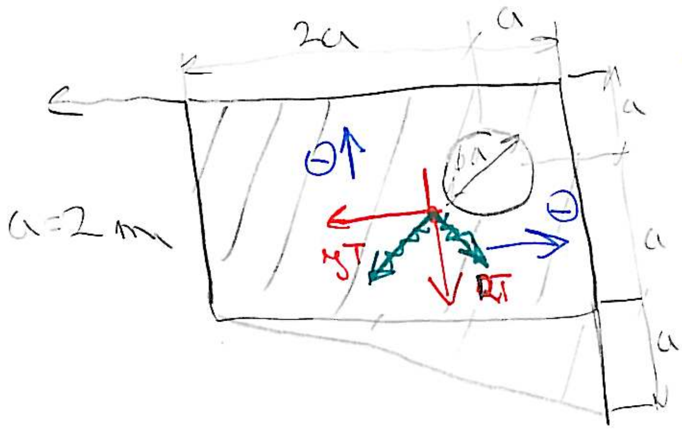
1



$$T: y_T = \frac{\left(10{,}5 - \frac{\pi}{4}\right)}{\left(7{,}5 - \frac{\pi}{4}\right)} \cdot a = 2{,}89\ m\ (=1{,}44a)$$

$$z_T = \frac{9{,}5 - \frac{\pi}{4}}{\left(7{,}5 - \frac{\pi}{4}\right)} \cdot a = 2{,}6\ m\ (=1{,}3a)$$

$$* \; J_y = \frac{585}{36} a^4 - \frac{17\pi a}{64} = 246{,}65\ m^4$$

$$J_z = \frac{81}{4} a^4 - \frac{17\pi a}{64} = 310{,}65\ m^4$$

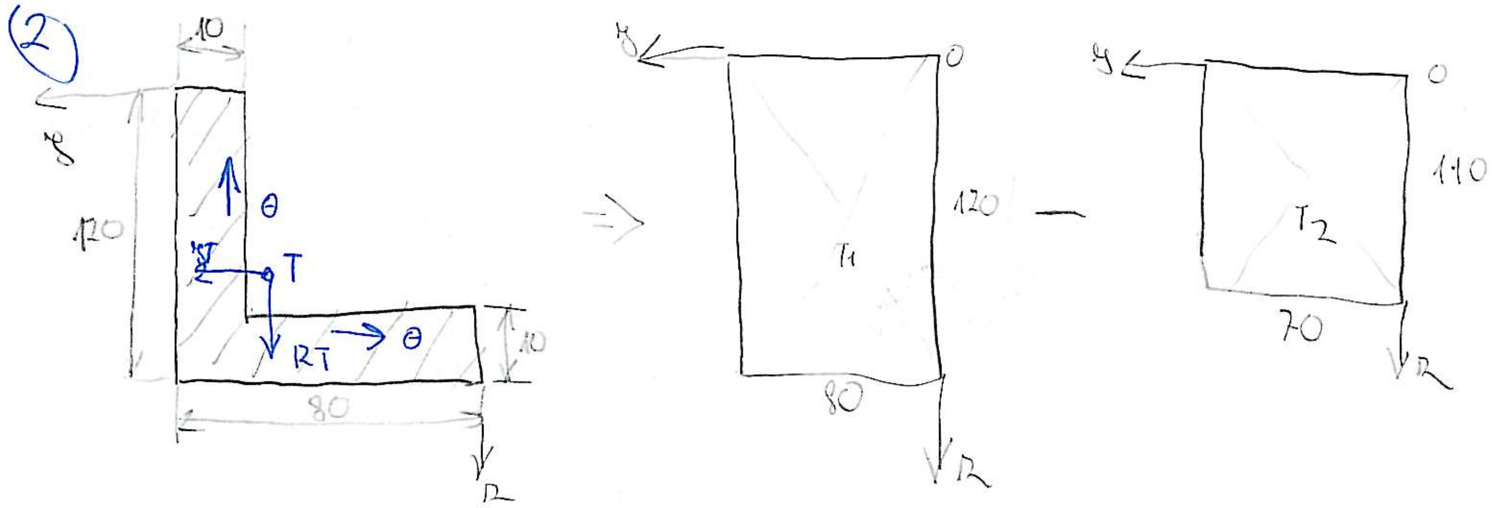
$$J_{yz} = \frac{99}{8} a^4 - \frac{\pi a^4}{4} = 185{,}4\ m^4$$

$$J_{yT} = J_y - N_y^2 \cdot S = \frac{585}{36} a^4 - \frac{17\pi a}{64} - (-2{,}89)^2 \cdot \left(7{,}5 - \frac{\pi}{4}\right) \cdot a^2 =$$
$$= 246{,}65 - 224{,}32 = \underline{\underline{22{,}33\ m^4}}$$

$$J_{zT} = J_z - N_z^2 \cdot S = \frac{81}{4} a^4 - \frac{17\pi}{64} - (-2{,}6)^2 \cdot \left(7{,}5 - \frac{\pi}{4}\right) \cdot a^2 =$$
$$= 310{,}65 - 181{,}56 \doteq \underline{\underline{129{,}1\ m^4}}$$

$$J_{y_T z_T} = J_{yz} - (-\cancel{No}\,N_y) \cdot (-N_z) \cdot S = \frac{99}{8} a^4 - \frac{\pi a^4}{4} - (-2{,}89) \cdot$$
$$(-2{,}6) \cdot \left(7{,}5 - \frac{\pi}{4}\right) \cdot a^2 = 185{,}4 - 201{,}81 = \underline{\underline{-16{,}41\ m^4}}$$

(2)



$$y_T = \frac{\Sigma y_i \cdot \Sigma S_i}{\Sigma S_i} = \frac{40 \cdot 80 \cdot 120 - 35 \cdot 70 \cdot 110}{80 \cdot 120 - 70 \cdot 110} = 60{,}26 \text{ mm}$$

$$z_T = \frac{\Sigma z_i \cdot \Sigma S_i}{\Sigma S_i} = \frac{60 \cdot 80 \cdot 120 - 55 \cdot 70 \cdot 110}{80 \cdot 120 - 70 \cdot 110} = 80{,}26 \text{ mm}$$

$$J_y = \frac{bh^3}{3} \Rightarrow \frac{80 \cdot 120^3}{3} - \frac{70 \cdot 110^3}{3} = 15023333{,}33 \text{ mm}^4$$

$$J_z = \frac{hb^3}{3} \Rightarrow \frac{120 \cdot 80^3}{3} - \frac{110 \cdot 70^3}{3} = 7903333{,}33 \text{ mm}^4$$

$$J_{yz} = \frac{b^2h^2}{4} \Rightarrow \frac{120^2 \cdot 80^2}{4} - \frac{110^2 \cdot 70^2}{4} = 8217500 \text{ mm}^4$$

$$J_{yT} = J_y - N_y^2 \cdot S = J_y - z_T^2 \cdot S = 15023333{,}33 - (-80{,}26)^2 \cdot (120 \cdot 8 - 110 \cdot 70)$$
$$= \underline{2784104{,}9 \text{ mm}^4}$$

$$J_{zT} = J_z - N_z^2 \cdot S = J_z - y_T^2 \cdot S = 7903333{,}33 - (-60{,}26)^2 \cdot (120 \cdot 80 - 110 \cdot 70)$$
$$= \underline{1003924{,}9 \text{ mm}^4}$$

$$J_{yTzT} = J_{yz} - y_T \cdot z_T \cdot S = 8217500 - (-(60{,}26) \cdot (-80{,}26) \cdot (120 \cdot 80 - 110 \cdot 70))$$
$$= \underline{-971788{,}4 \text{ mm}^4}$$

(3) $R = 50\,mm,\ r = 30\,mm$

$$y_T = \frac{\sum y_i \cdot \sum S_i}{\sum S_i} = 0\,mm$$

$$z_T = \frac{\sum z_i \cdot \sum S_i}{\sum S_i} = \frac{\frac{4}{3}\frac{R}{\pi}\cdot\frac{\pi R^2}{2} - \frac{4}{3}\cdot\frac{r}{\pi}\cdot\frac{\pi r^2}{2}}{\frac{\pi R^2}{2} - \frac{\pi r^2}{2}} = \frac{\frac{4}{3}\cdot\frac{R^3}{2} - \frac{4}{3}\cdot\frac{r^3}{2}}{\frac{\pi}{2}\cdot(R^2 - r^2)}$$

$$= \frac{2}{3}\cdot(R^3 - r^3)\cdot\frac{2}{\pi}\cdot\frac{1}{(R^2 - r^2)} = \frac{2}{\pi}\cdot\frac{1}{(R^2 - r^2)} = \frac{4}{3\pi}\cdot\frac{R^3 - r^3}{R^2 - r^2} =$$

$$= 26\,mm$$

$$J_p = \iint_A r^2\,dS = \int_0^R r^2\cdot\pi r\,dr = \frac{\pi R^4}{4}$$

$$J_p = J_y + J_z \Rightarrow J_y = J_z = \frac{J_p}{2} = \frac{\pi R^4}{8}$$

$$J_y = \frac{\pi R^4}{8} \Rightarrow \frac{\pi\cdot R^4}{8} - \frac{\pi r^4}{8} - \frac{\pi\cdot r^4}{8} = \frac{\pi}{8}\cdot(R^4 - r^4) = \frac{\pi}{8}\cdot(50^4 - 30^4)$$

$$= 2\,136\,283\,mm^4$$

$$J_z = J_y = 2\,136\,283\,mm^4$$

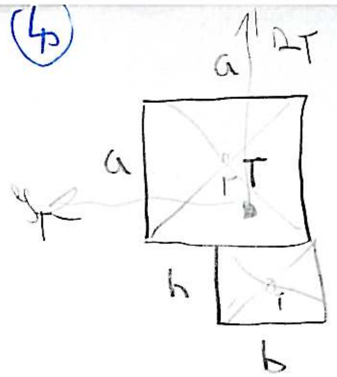
$J_{yz} = 0\,mm^4$ – objekt je souměrný dle osy z

$$J_{yT} = J_y - z_T^2\cdot S = J_y - z_T^2\cdot S = 2\,136\,283 - (-26^2)\cdot\left(\frac{\pi}{2}\cdot(R^2 - r^2)\right) = 437\,309{,}7\,mm^4$$

$$J_{zT} = J_z - y_T^2\cdot S = J_z - y^2\cdot S = 2\,136\,283 - 0 = 2\,136\,283\,mm^4$$

$$J_{y_T z_T} = J_{yz} - y_T\cdot z_T\cdot S = 0 - 0 = 0\,mm^4$$

(4b)



$a = 100$, $b = 50$, $h = 80$

$$y_T = \frac{0 \cdot a^2 + 25 \cdot 80}{100^2 + 50 \cdot 80} = 7{,}14 \text{ mm}$$

$$z_T = \frac{0 \cdot a^2 + (50+40) \cdot 50 \cdot 80}{100^2 + 50 \cdot 80} = 25{,}71 \text{ mm}$$

$$J_{yT} = \frac{a^4}{12} - (z_T)^2 \cdot a^2 + \frac{h^3 b}{12} + \left(\frac{a}{2} + \frac{h}{2} - z_T\right)^2 \cdot h \cdot b =$$

$$= \frac{100^4}{12} - 25{,}71^2 \cdot 100^2 + \frac{80^3 \cdot 50}{12} + (50 - 25{,}72)^2 \cdot 80 \cdot 50$$

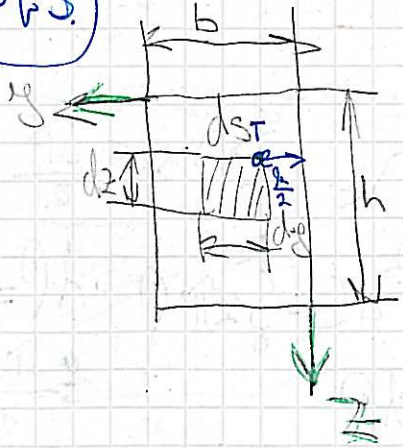
$$= 20389442 \text{ mm}^4$$

$$J_{zT} = \frac{a^4}{12} + y_T^2 \cdot a^2 + \frac{b^3 h}{12} - \left(\frac{b}{2} - y_T\right)^2 \cdot h \cdot b = \frac{100^4}{12}$$

$$+ 7{,}14^2 \cdot 100^2 + \frac{50^3 \cdot 80}{12} + (-25 + 7{,}14)^2 \cdot 80 \cdot 50 = 8400544 \text{ mm}^4$$

$$J_{yTz} = J_{yz} + a \cdot b \cdot s = -6428572 \text{ mm}^4$$

Př 5.



$$U_y = \int z \cdot dS = \int_0^b \int_0^h z \, dz \, dy = \int_0^b \left[\frac{z^2}{2}\right]_0^h dy$$

$$= \frac{h^2}{2} \cdot b$$

$$U_z = \int_S y \cdot dS = \int_0^h \int_0^b y \, dy \, dz = \int_0^h \left[\frac{y^2}{2}\right]_0^b dz =$$

$$= \frac{b^2}{2} \cdot h$$

$$J_y = \int z^2 dS = \int_0^b \int_0^h z^2 \, dz \, dy =$$

$$= \int_0^b \left[\frac{z^3}{3}\right]_0^h dy = \frac{h^3}{3} \cdot b$$

$$\int_0^b dy = [y]_0^b \qquad J_z = \frac{b^3 h}{3}$$

$$J_{yz} = \int yz \, dS = \int_0^b \int_0^h yz \, dz \, dy = \int_0 \frac{z^2}{2} dz = \left[\frac{z^2}{2}\right]_0^h \cdot \left[\frac{y^2}{2}\right]_0^b$$

$$= \frac{h^2 \cdot b^2}{4}$$

$$J_{y'} = J_{yT} + a^2 S$$

$$J_{yT} = I_{y'} - a^2 S = \frac{h^3 b}{3} - \frac{h^2}{4} \cdot hb =$$

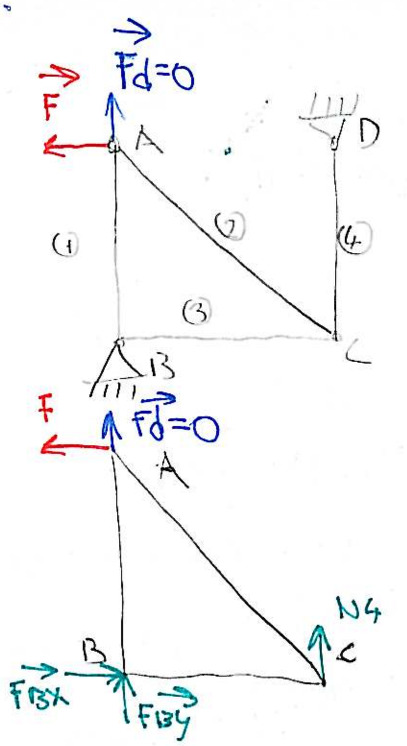
$$= \frac{4h^3 b - 3hb}{12} = \frac{1}{12} h^3 b$$

$$J_{zT} = \frac{1}{12} h b^3$$

$$J_{yzT} = 0$$

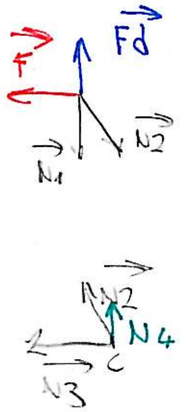
(1)

Castiglian 10 příkladů



$s_{ex} = n_{ex} - \nu = 3 - 3 = 0$

$s_{in} = 3 - (2\cdot 3 - 3) = 0$



$$F - N_2 \cdot \frac{\sqrt{2}}{2} = 0$$
$$F_d - N_1 - N_2 \cdot \frac{\sqrt{2}}{2} = 0$$

$N_2 = \sqrt{2}\cdot F$

$N_1 = F_d - F$

$$N_3 + N_2 \cdot \frac{\sqrt{2}}{2} = 0$$
$$N_4 + N_2 \cdot \frac{\sqrt{2}}{2} = 0$$

$N_3 = -F$

$N_4 = -F$

$$u_A = \frac{\partial W}{\partial F} = \sum_{i=1}^{4} \frac{N_i \cdot l_i}{E_i \cdot S_i} \cdot \frac{\partial N_i}{\partial F} = \frac{1}{ES} \cdot \left[(F_d - F)\cdot a \cdot (-1) + (\sqrt{2}\cdot F)\cdot a \cdot \sqrt{2} \cdot \sqrt{2} + (-F)\cdot a \cdot (-1) + (-F)\cdot a \cdot (-1) \right] = |F_d = 0| =$$

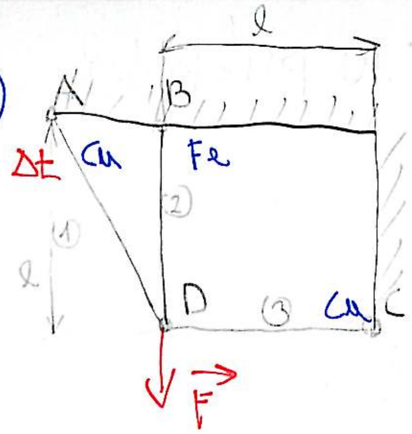
$$\left[-300\,000 \cdot 2 \cdot (-1) + \sqrt{2} \cdot 300\,000 \cdot 2 \cdot \sqrt{2} \cdot \sqrt{2} + (-300\,000) \cdot 2\,(-1) \right] \cdot \frac{1}{2\cdot 10^5 \cdot 1256{,}64} \doteq 13{,}9\ \text{mm}$$

~~2,4 mm~~

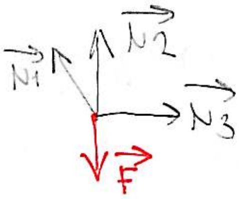
$$v_A = \frac{\partial W}{\partial F_d} = \frac{1}{ES}\left[(F_d - F)\cdot a \cdot (+1) + (\sqrt{2}\cdot F)\cdot a\sqrt{2}\cdot 0 + 0 + 0 \right] = |F_d = 0| = \frac{-Fa}{ES} = \frac{-300\,000\cdot 2}{2\cdot 10^5 \cdot 1256{,}64} \doteq -2{,}4\ \text{mm}$$

$$\sigma_{max} = \frac{\sqrt{2}\cdot F}{\frac{\pi d^2}{4}} = \frac{\sqrt{2}\cdot 300\,000}{1256{,}64} \doteq \underline{337{,}62\ \text{MPa}} < \sigma_k \ (\sigma_k = 400\ \text{MPa})$$

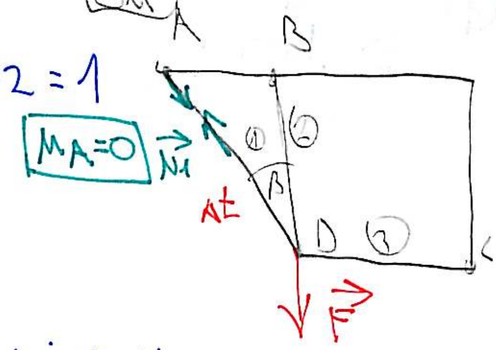
(2)



Cu



Cu



$S = \mu - \nu = 3 - 2 = 1$

SR: $-N_1 \cdot \sin\beta + N_3 = 0$

$N_1 \cos\beta + N_2 - F = 0$

$N_3 = N_1 \sin\beta = -11{,}9\,kN$

$N_2 = F - N_1 \cdot \cos\beta = 120{,}6\,kN$

$$M_A = \frac{\partial W}{\partial N_1} = \frac{1}{S} \cdot \left[\frac{N_1 \cdot \frac{l}{\cos\beta}}{E_{Cu}} \cdot 1 + \frac{(F - N_1 \cdot \cos\beta) \cdot l}{E_{Fe}} \cdot (-\cos\beta) + \frac{N_1 \cdot \sin\beta \cdot l}{E_{Cu}} \cdot \sin\beta \right] + \frac{l}{\cos\beta} \cdot \alpha_{Cu} \cdot \Delta t = 0 \Rightarrow N_1$$

$$l = 1: \quad 0 = \frac{1}{S} \cdot \left[\frac{N_1}{E_{Cu} \cdot \cos\beta} + \frac{-F \cdot \cos\beta + N_1 \cos^2\beta}{E_{Fe}} + \frac{N_1 \sin^2\beta}{E_{Cu}} \right] + \frac{\alpha_{Cu} \cdot \Delta t}{\cos\beta} \quad / \cdot S \quad / \cdot E_{Cu} \cdot E_{Fe} \cdot \cos\beta$$

$$N_1 = \frac{F \cdot \cos^2\beta \cdot E_{Cu} \cdot \alpha_{Cu} \cdot \Delta t \cdot S \cdot E_{Cu} \cdot E_{Fe}}{E_{Fe} + E_{Cu} \cdot \cos^3\beta + E_{Fe} \cdot \sin^2\beta \cdot \cos\beta} = -23797{,}8\,N \doteq 23{,}8\,kN$$

$$k_k^{Cu} = \frac{\sigma_k^{Cu}}{\sigma_{max}^{Cu}} \qquad k_k^{Fe} = \frac{\sigma_k^{Fe}}{\sigma_{max}^{Fe}} \qquad k_k^{Cu} = \frac{130}{34} \doteq 3{,}8 \qquad k_k^{Fe} = \frac{350}{172{,}3} \doteq 2{,}03$$

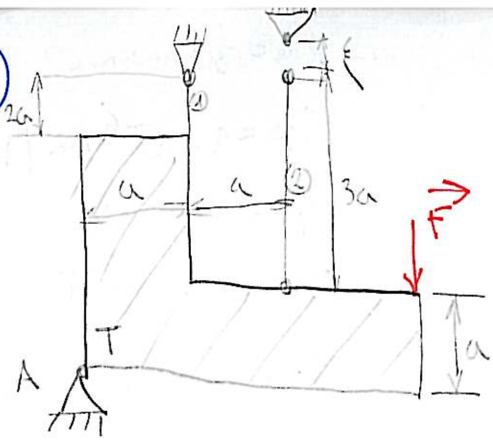
$$\sigma_{Cu} = \frac{\max\{N_1, N_3\}}{S} \qquad \sigma_{max}^{Fe} = \frac{N_2}{S} \qquad \sigma_{max}^{Cu} = \frac{N_1}{S} \doteq \frac{23800}{0{,}0007} = 34\,000\,000 = 34\,MPa$$

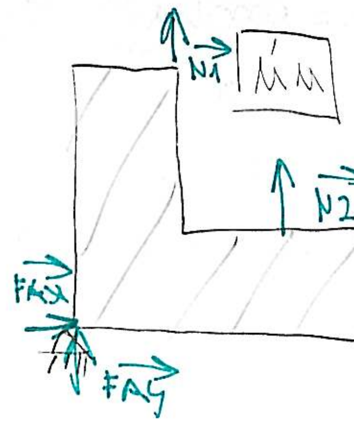
$$\sigma_{max}^{Fe} = \frac{N_2}{S} = \frac{120\,600}{0{,}0007} =$$

$$= 172\,285\,714{,}3\,Pa \doteq 172{,}3\,MPa$$

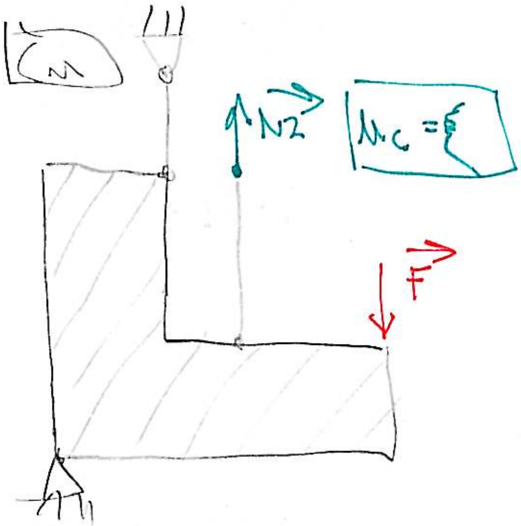
$$k_k = \min\{k_k^{Cu}, k_k^{Fe}\} \Rightarrow k_k = k_k^{Fe} = 2{,}03$$

(3)





$S = n - v = 4 - 3 = 1$



SR: $F_{Ax} = 0$

$F_{Ay} + N_1 + N_2 - F = 0$

$N_1 a + N_2 \cdot 2a - F \cdot 4a = 0 \rightarrow N_1 = 4F - 2N_2 = 1{,}3F - 19\,000$

$$N_c = \frac{\partial W}{\partial N_2} = \sum_{i=1}^{2} \frac{N_i l_i}{E_i S_i} \cdot \frac{\partial N_i}{\partial N_2} = \frac{1}{E} \cdot \left[\frac{4F - 2N_2}{S_1} \cdot 2a \cdot (-2) + \frac{N_2}{S_2} \cdot 3a \cdot 1 \right] = \xi \rightarrow N_2 = 9400 + 1{,}4 \cdot F$$

$$= \frac{1}{E} \cdot \left[\frac{4F - 2\cdot(4F - 2N_2)}{S_1} \cdot (-4a) + \frac{N_2 \cdot 3a}{S_2} \right] = \frac{1}{E} \cdot$$

$$\left[\frac{4N_2 - 4F}{\frac{\pi d^2}{4}} \cdot (-4a) + \frac{N_2 \cdot 3a}{h \cdot c} \right] = \xi$$

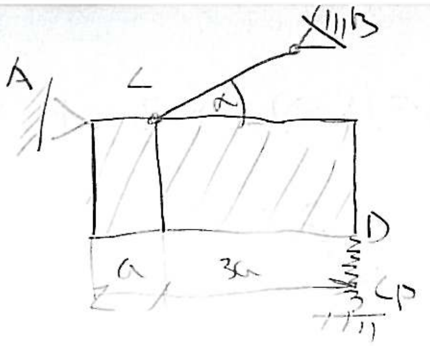
$$\sigma^{(1)} = \frac{N_1}{S_1} = 3600F - 60 \cdot 10^6$$

$$\sigma^{(2)} = \frac{N_2}{S_2} = (31 + 0{,}005 \cdot F) \cdot 10^6$$

$$k_k^{(1)} = \frac{\sigma_k}{\sigma^{(1)}} \Rightarrow F^{(1)} = 72\ kN$$

$$k_k^{(2)} = \frac{\sigma_k}{\sigma^{(2)}} \Rightarrow F^{(2)} = 35\ kN$$

(4)



$m = 5000\,kg$

$\alpha = 30^\circ$

$l = 1{,}7\,m$

$a = 500\,mm = 0{,}5\,m$

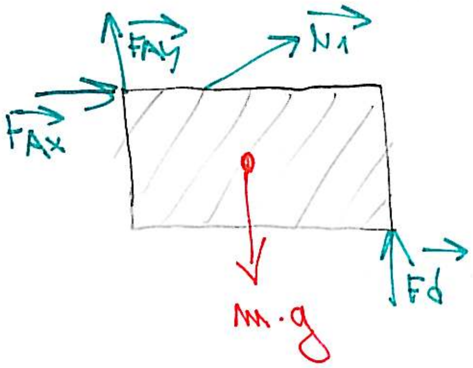
$\phi d = 30\,mm$

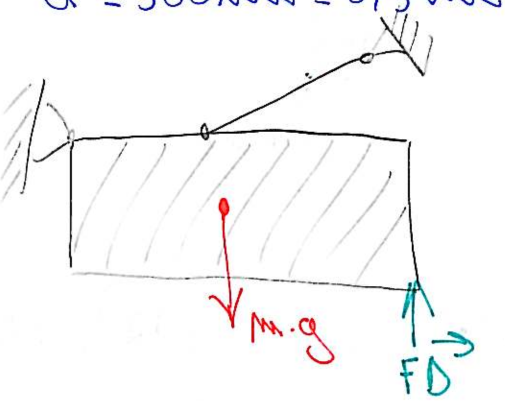
$C_p = 1 \cdot 10^{-6}\,m/N$

$g = 10\,m/s$

$E = 2 \cdot 10^5\,MPa$

$\sigma_k = 300\,MPa$





$S = \mu \cdot \nu = 4 - 3 = 1$

$\boxed{w_D = F_D \cdot C_p}$

$\boxed{SR:}$

$F_{Ax} + N_1 \cdot \cos\alpha = 0$

$F_{Ay} - m \cdot g + F_D + N_1 \sin\alpha = 0$

$N_1 \sin\alpha \cdot a - m \cdot g \cdot 2a + F_D \cdot 4a = 0 \Rightarrow N_1 = m \cdot g \cdot \frac{2}{\sin\alpha}$

$- F_D \cdot \frac{4}{\sin\alpha} = 4mg - 8F_D = 4 \cdot 5000 \cdot 10 - 8 \cdot 10872{,}5 =$

$= 113020\,N \doteq 113\,kN$

$w_D = \frac{\partial W}{\partial F_D} = \frac{N_i \cdot l_i}{E_i \cdot S_i} \cdot \frac{\partial N_i}{\partial F_D} = \frac{4mg - 8F_d}{E \cdot S} \cdot l\,(-8) = -F_d \cdot C_p \rightarrow F_d$

$(4mg - 8F_D) \cdot (-8l) = -F_D \cdot C_p \cdot ES$

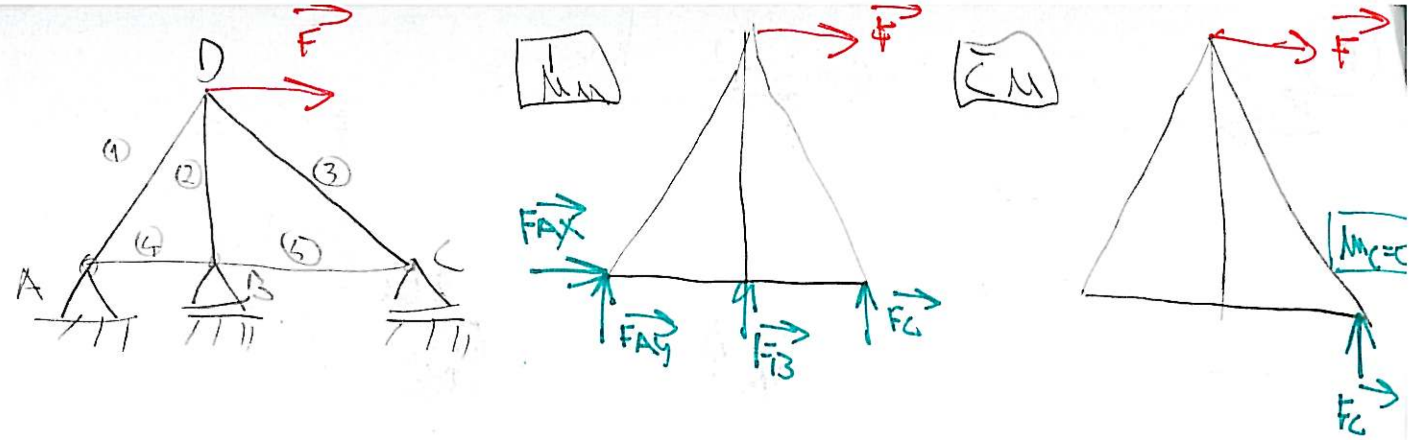
$-32mgl + 64F_dl = -F_D \cdot C_p \cdot ES$

$64F_D \cdot l + F_D \cdot C_p \cdot ES = 32mgl \Rightarrow F_D = \frac{32mg \cdot l}{64l + C_p \cdot ES}$

$= \frac{32 \cdot 5000 \cdot 10 \cdot 1{,}7}{64 \cdot 1{,}7 + 1 \cdot 10^{-6} \cdot 2 \cdot 10^{11} \cdot \frac{\pi \cdot 0{,}03^2}{4}} = 10872{,}5\,N \doteq \underline{\underline{10{,}9\,kN}}$

$\sigma^{(1)} = \frac{N_1}{S_1} = \frac{113020}{\frac{\pi \cdot 0{,}03^2}{4}} \doteq 160\,MPa \quad k_k = \frac{300}{160} = \underline{\underline{1{,}875}}$

(5)

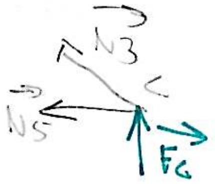


$a = 500\,mm = 0{,}5\,m$; $F = 10^5\,N$; $E = 2\cdot 10^5\,MPa$; $\sigma_k = 350\,MPa$

$\phi d = 30\,mm$

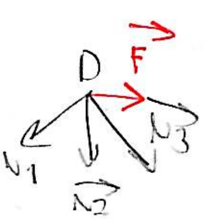
$s_{ex} = n_{ex} - D = 4 - 3 = 1$

$n_n = n - (2k-3) = 5 - (2\cdot 4 - 3) = 0$



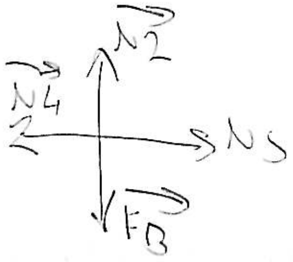
$\boxed{SR:}$ $N_3 = -\sqrt{2}\,F_C$

$N_5 = F_C$



$\boxed{SR:}$ $N_1 = \sqrt{2}\cdot F - \sqrt{2}\cdot F_C$

$N_2 = -F + 2F_C$



$\boxed{SR:}$ $N_4 = F_C$

$$w_C = \frac{\partial W}{\partial F_C} = \sum_{i=1}^{5} \frac{N_i l_i}{E_i \cdot S_i} \cdot \frac{\partial N_i}{\partial F_C} = \frac{1}{ES}\cdot\Big[(\sqrt{2}\cdot F - \sqrt{2}\cdot F_C)\cdot a\sqrt{2}\cdot(-\sqrt{2}) +$$
$$(-F + 2F_C)\cdot a\cdot 2 + (-\sqrt{2}\,F_C)\cdot a\cdot\sqrt{2}\cdot(-\sqrt{2}) + F_C\cdot a\cdot 1 + F_C\, a\cdot 1\Big]$$
$$= 0 \rightarrow F_C$$

$$\frac{1}{ES}\cdot\Big[(\sqrt{2}F - \sqrt{2}F_C)\cdot(-2a) + (-F + 2F_C)\cdot 2a + (-\sqrt{2}F_C)\cdot(-2a) +$$
$$2F_C\cdot a\Big] = 0$$

$$-\sqrt{2}F + \sqrt{2}F_C - F + 2F_C + \sqrt{2}F_C + F_C = 0$$

$F_C(\sqrt{2} + 2 + \sqrt{2} + 1) = F\cdot(1+\sqrt{2})$ $\quad F_C = \dfrac{F\cdot(1+\sqrt{2})}{2\cdot\sqrt{2}+3} = \dfrac{10^5(1+\sqrt{2})}{2\cdot\sqrt{2}+3} = \underline{41{,}4\,kN}$

$N_1 = \sqrt{2}\cdot F - \sqrt{2}\cdot F_C = \sqrt{2}\cdot 10^2 - \sqrt{2}\cdot 41{,}4 = 82{,}87\,kN$

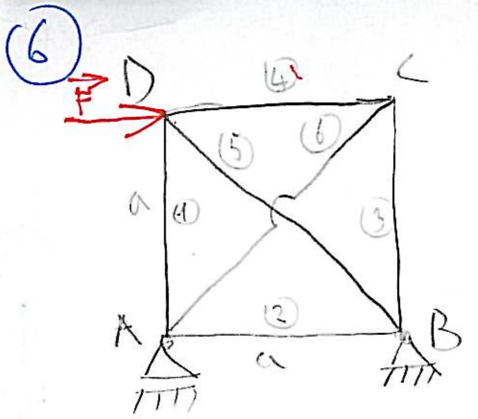
$N_2 = -F + 2F_C = -10^2 + 2\cdot 41{,}4 = -17{,}2\,kN$

$N_3 = -\sqrt{2}\cdot F = -\sqrt{2}\cdot 10^2 = -141{,}4\,kN \rightarrow \sigma_{max} = \dfrac{141400}{\frac{\pi\cdot 0{,}03^2}{4}} \doteq 200\,MPa$

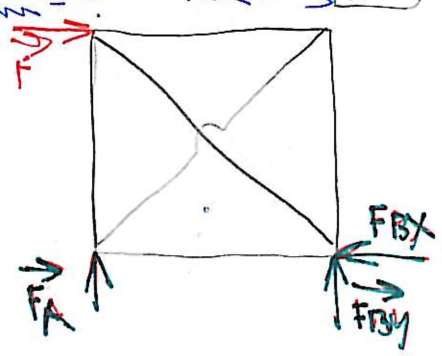
$N_4 = F_C = 41{,}4\,kN$

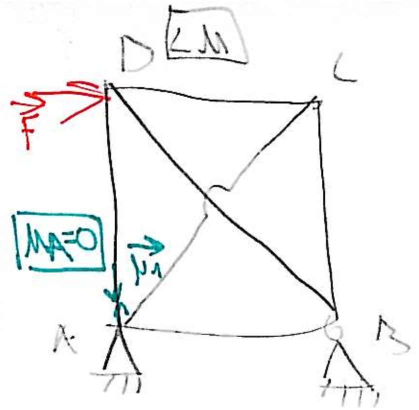
$N_5 = F_C = 41{,}4\,kN$ $\quad k_k = \dfrac{350}{200} = \underline{1{,}75}$

6



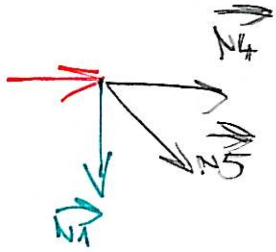
$d_{min} = ?$ $\quad k_k = 3$ [MM]





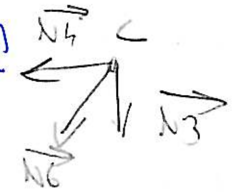
$n_{ex} = n_{ex} - \nu = 3 - 3 = 0$

$n_i = 6 - (8 - 3) = 1$



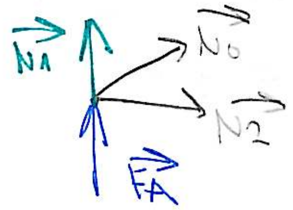
SR. $\quad N_5 = -N_1 \sqrt{2}$

$N_4 = N_1 - F$



SR $\quad N_6 = F\sqrt{2} - N_1 \sqrt{2}$

$N_3 = N_1 - F$



SR $\quad N_2 = N_1 - F$

$$M_A = \frac{\partial W}{\partial N_1} = \sum_{i=1}^{6} \frac{N_i l_i}{E_i S_i} \cdot \frac{\partial N_i}{\partial N_1} = \frac{1}{\cancel{ES}} \cdot \Big[N_1 \cdot a \cdot 1 \cdot 1 + (N_1 - F) \cdot a \cdot 1 \cdot 3 +$$
$$(-N_1\sqrt{2}) \cdot a \cdot \sqrt{2} \cdot (-\sqrt{2}) + (F \cdot \sqrt{2} - N_1 \cdot \sqrt{2}) \cdot a\sqrt{2} \cdot (-\sqrt{2}) \Big] =$$
$$0 \rightarrow N_1$$

$$N_1 \cdot \cancel{a} + 3N_1 \cancel{a} - 3F \cdot \cancel{a} + 2\sqrt{2} N_1 \cancel{a} - 2\sqrt{2} F \cdot \cancel{a} + 2\sqrt{2} \cdot N_1 \cdot \cancel{a} = 0$$
$$4N_1 + 4\sqrt{2} N_1 - 3F - 2\sqrt{2} F = 0$$

$$N_1 = \frac{3 \cdot F + 2\sqrt{2} F}{4 + 4\sqrt{2}} = \frac{3 \cdot 10^5 + 2 \cdot \sqrt{2} \cdot 10^5}{4 \cdot (1 + \sqrt{2})} = 60{,}4\,kN$$

$N_2 = N_1 - F = 60{,}4 - 100 = -39{,}6\,kN$

$N_3 = N_1 - F = -39{,}6\,kN$

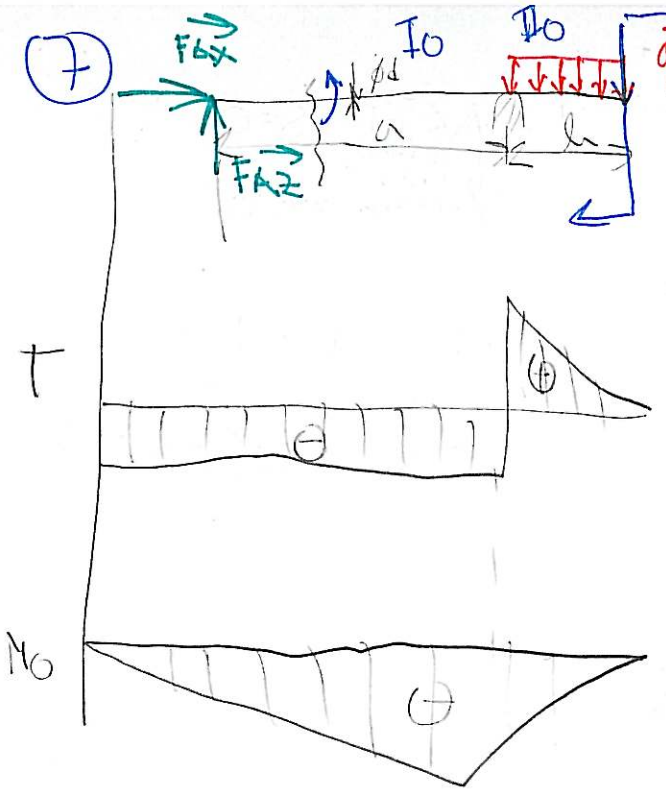
$N_4 = N_1 - F = -39{,}6\,kN$

$N_5 = -N_1 \cdot \sqrt{2} = -85{,}42\,kN \rightarrow N_{max} \rightarrow \sigma_{max} = \frac{N_5 \cdot 4}{\pi d^2} = -108\,76 \cdot d^{-2}\,Pa$

$N_6 = F \cdot \sqrt{2} - N_1 \cdot \sqrt{2} = 56\,kN$

$$k_k = \frac{\sigma_k}{\sigma_{max}} \Rightarrow |\sigma_{max}| = \frac{\sigma_k}{k_k}$$

$$d = \sqrt{\frac{k_k}{\sigma_k} \cdot 108\,760} = 0{,}031\,m = 31\,mm$$



$w_C = ?!$ $\varphi_C = ?!$

$\boxed{SR:}$ $F_{AX} = 0$

$F_{AZ} + F_B - q \cdot h - F_d = 0$

$F_{AZ} \cdot a + \frac{q \cdot h^2}{2} + F_d \cdot h + M_d = 0$

$F_{AZ} = -\frac{q \cdot h^2}{2a} - \frac{h}{a} \cdot F_d - \frac{1}{a} \cdot M_d$

$F_B = F_d + q \cdot h + \frac{q \cdot h^2}{2} + \frac{h}{a} \cdot F_d + \frac{1}{a} \cdot M_d = q \cdot h + \frac{q \cdot h}{2a} + F_d \cdot \left(1 + \frac{h}{a}\right) + \frac{1}{a} + M_d$

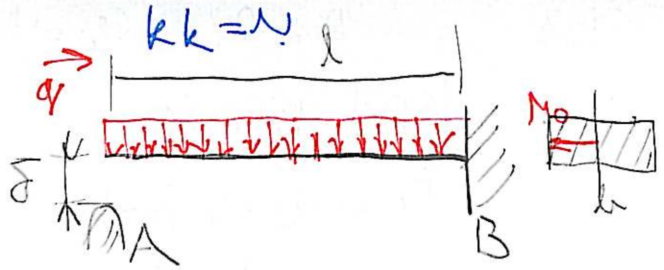
$$w_C = \frac{\partial W}{\partial F_d} = \int \frac{M_o}{EJ} \cdot \frac{\partial M_o}{\partial F_d} \, dx = \frac{1}{EJ} \cdot \left[\int_0^a \left(-\frac{q \cdot h^2}{2a} \cdot x\right) \cdot \left(-\frac{h}{a} \cdot x\right) dx + \int_0^h \left(-\frac{q \cdot x^2}{2}\right) \cdot (-x)\, dx \right] = \frac{1}{EJ} \cdot \left[\frac{q \cdot h^3}{2a^2} \cdot \frac{x^3}{3}\right]_0^a + \left[\frac{q}{2} \cdot \frac{x^4}{4}\right]_0^h = \frac{1}{E} \cdot \frac{64}{\pi d^4} \cdot \left(\frac{q \cdot h^3 \cdot a^3}{6 \cancel{a^2}} + \frac{q \cdot h^4}{8}\right) = \frac{64}{E \pi d^4} \cdot \left(\frac{4 q h^3 a + 3 q h^4}{24}\right) = \frac{64}{2 \cdot 10^{11} \cdot \pi \cdot 0{,}03^4} \cdot \left(\frac{4 \cdot 30 \cdot 2^3 \cdot 4 + 3 \cdot 30 \cdot 2^4}{24}\right)$$

$$= 0{,}02767\,m \doteq \underline{\underline{28\,mm}}$$

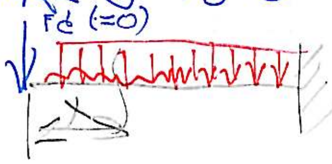
$$\varphi_C = \frac{\partial W}{\partial M_d} = \frac{1}{EJ} \cdot \left[\int_0^a \left(-\frac{q \cdot h^2}{2a} \cdot x\right) \cdot \left(-\frac{x}{a}\right) dx + \int_0^h \left(-\frac{q \cdot x^2}{2}\right) \cdot (-1)\, dx \right] = \frac{1}{EJ} \cdot \left[\left[\frac{q \cdot h^2}{2a^2} \cdot \frac{x^3}{3}\right]_0^a + \left[\frac{q}{2} \cdot \frac{x^3}{3}\right]_0^h \right] = \frac{1}{EJ} \cdot \left(\frac{q \cdot h^2 \cdot a^3}{6 \cancel{a^2}} + \frac{q \cdot h^3}{6}\right) = \frac{1}{EJ} \cdot \left(\frac{q \cdot h^2 \cdot a + q \cdot h^3}{6}\right) = \frac{64}{E \cdot \pi \cdot d^4} \cdot \left(\frac{q \cdot h^2 \cdot (a+h)}{6}\right)$$

$$= \frac{64}{2 \cdot 10^{11} \cdot \pi \cdot 0{,}03^4} \cdot \left(\frac{30 \cdot 2^2 \cdot \cancel{6}}{\cancel{6}}\right) = 0{,}0151\,rad \Rightarrow \underline{\underline{0^\circ 52'}}$$

(8)



a) $w_A < \delta$: S = 0



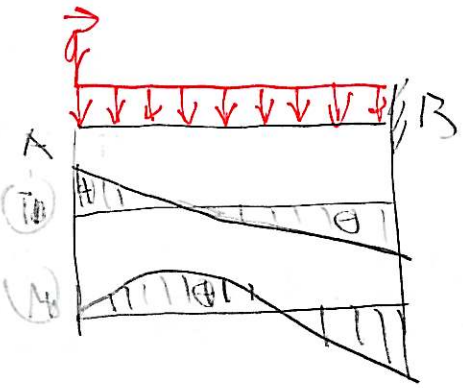
$$M_O = -F_d \cdot x - \frac{q \cdot x^2}{2}$$

$$w_A = \frac{\partial W}{\partial F_d} = \int_l \frac{M_O}{EJ} \cdot \frac{\partial M_O}{\partial F_d} \cdot dx = \frac{1}{EJ} \cdot \int_0^l \left(-F_d \cdot x - \frac{q \cdot x^2}{2}\right) \cdot (-x)\, dx = |F_d = 0| = \frac{1}{EJ} \int_0^l \left(-\frac{q \cdot x^2}{2}\right)$$

$$\cdot (-x) \cdot dx = J = \frac{bh^3}{12} \;\# > \delta$$

b) $w_A > \delta$: S = 1

$\boxed{w_A = \delta}$



$$M_O = F_A \cdot x - \frac{q \cdot x^2}{2}$$

$$w_A = \frac{\partial W}{\partial F_A} = \frac{1}{EJ} \cdot \int_0^l \left(F_A x - \frac{q x^2}{2}\right) \cdot x \cdot dx = -\delta \to F_A$$

$$\frac{1}{EJ} \cdot \left[F_A \frac{x^3}{3} - \frac{q}{2} \cdot \frac{x^4}{4}\right]_0^l = -\delta$$

$$F_A \cdot \frac{l^3}{3} - \frac{q}{2} \cdot \frac{l^4}{4} = -\delta \cdot EJ$$

$$8 F_A \cdot l^3 - 3q \cdot l^4 = -24\delta \cdot EJ$$

$$F_A = \frac{3ql^4 - 24\delta \cdot EJ}{8l^3} = \frac{3q \cdot l^4 - 24 \cdot \delta \cdot E \cdot \frac{bh^3}{12}}{8l^3}$$

$$= 76 N$$

$$M_O(x = l) = 76 \cdot 1{,}75 - \frac{150 \cdot 1{,}75^2}{2} \doteq -69{,}7 Nm$$

$$\frac{dM_O}{dx} = F_A - q \cdot x = 0 \to x = 0{,}507 m$$

$$M_O(x = 0{,}507 m) = 76 \cdot 0{,}507 - \frac{150 \cdot 0{,}507^2}{2} = 19{,}25 Nm$$

$$\sigma_{max} = \frac{M_{O max}}{W_O} = \frac{M_{O max} 6}{bh^2}$$

$$W_O = \frac{\frac{bh^3}{12}}{\frac{h}{2}} = \frac{bh^2}{6}$$

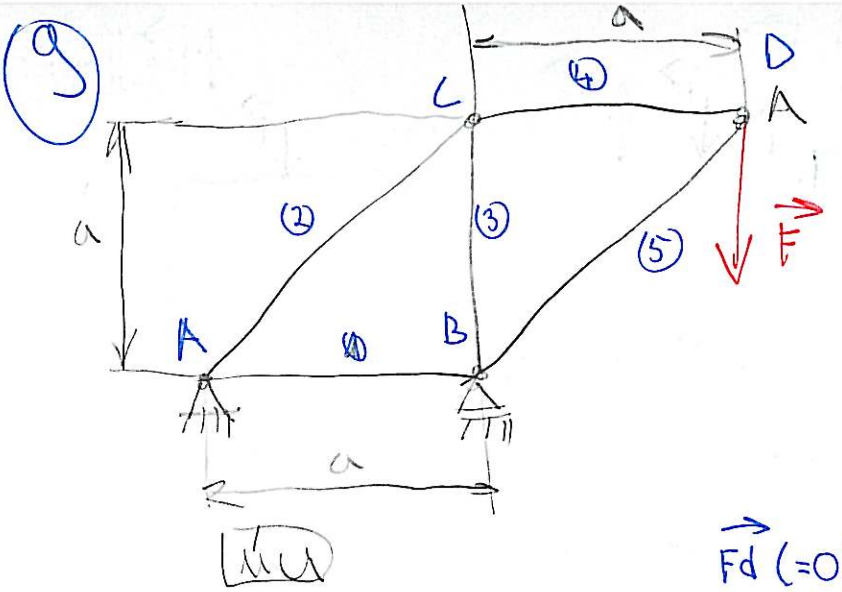
$$\sigma_{max} = \frac{-96{,}7 \cdot 6}{0{,}03 \cdot 0{,}02^2} = -48{,}35 MPa$$

$$k_k = \frac{\sigma_k}{|\sigma_{max}|} = \frac{200}{48{,}35} = \underline{4{,}14}$$

podm. linearity

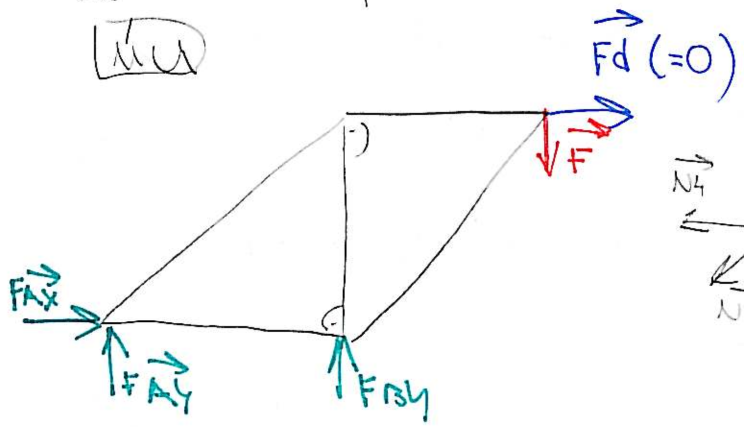
$$|\sigma_{max}| \leq \sigma_k$$

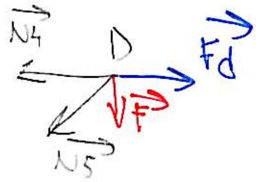
$$\underline{\underline{48{,}35 < 200 \; [MPa]}}$$



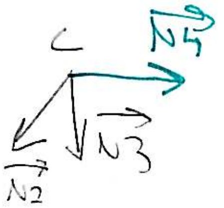
$S_{ex} = \mu_{ex} - \nu = 3 - 3 = 0$

$5 - (2 \cdot 4 - 3) = 0$

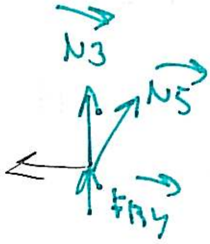




$$Fd - N_4 - N_5 \cdot \frac{\sqrt{2}}{2} = 0$$
$$-F - N_5 \cdot \frac{\sqrt{2}}{2} = 0$$
$$N_5 = -\sqrt{2} \cdot F$$
$$N_4 = Fd + F$$



$$N_4 - N_2 \cdot \frac{\sqrt{2}}{2} = 0$$
$$-N_3 - N_2 \cdot \frac{\sqrt{2}}{2} = 0$$
$$Fd + F - N_2 \cdot \frac{\sqrt{2}}{2} = 0$$
$$N_2 = \sqrt{2}\,(Fd + F)$$
$$N_3 = -Fd - F$$

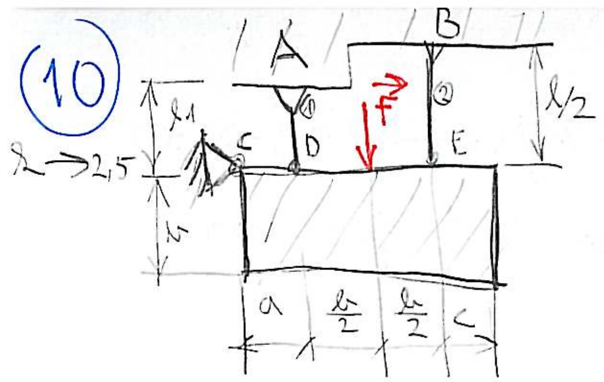


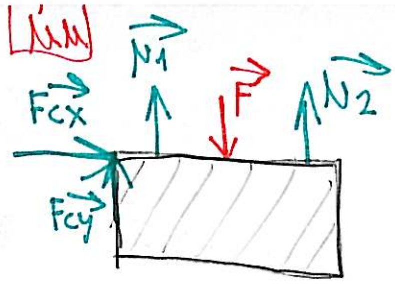
$$N_5 \cdot \frac{\sqrt{2}}{2} - N_1 = 0$$
$$N_3 + F_{By} = 0$$
$$N_1 = -F$$
$$(F_{By} = Fd + F)$$

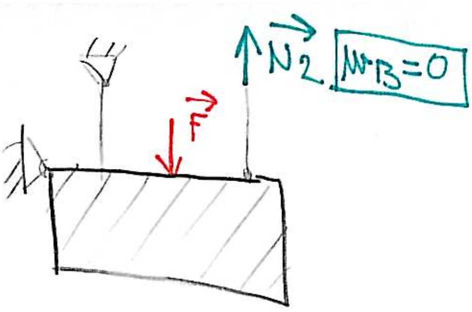
$$N_A = \frac{\partial W}{\partial Fd} = \frac{1}{ES} \cdot \Big[(\sqrt{2}(Fd+F)) \cdot a \cdot \sqrt{2} \cdot \sqrt{2} + (-Fd - F) \cdot a \cdot (-1) + (Fd + F) \cdot a \cdot 1 \Big] = \sqrt{2} \cdot F \cdot 2a \cdot F \cdot a + F \cdot a = 2 \cdot \sqrt{2} \cdot Fa + 2 \cdot Fa =$$
$$= F \cdot a \cdot (2 \cdot \sqrt{2} + 2)] / ES$$

$$N_A = \frac{\partial W}{\partial F} = \frac{1}{ES} \cdot \Big[(-F) \cdot a \cdot (-1) + (\sqrt{2}(Fd + F) \cdot a \sqrt{2} \cdot \sqrt{2} + (Fd \cdot F) \cdot a \cdot (-1) + (F + F) \cdot a \cdot 1 + (-\sqrt{2} \cdot F) \cdot a \sqrt{2} \cdot (\sqrt{2}) \Big] =$$
$$= F + 2\sqrt{2}\, F \cdot a + Fa + F \cdot a + 2\sqrt{2}\, F \cdot a = \boxed{[F \cdot a \cdot (3 + 4\sqrt{2})] / ES}$$

(10)







$$s = \mu - \nu = 4 - 3 = 1$$

$\boxed{SR}$ $F_{Cx} = 0$

$$F_{Cy} + N_1 - F + N_2 = 0$$

$$N_1 \cdot a - F \cdot \left(a + \frac{b}{2}\right) + N_2 \cdot (a + b) = 0 \rightarrow N_1 = \frac{F\left(a + \frac{b}{2}\right) - N_2 \cdot (a+b)}{a}$$

$$= \frac{F \cdot 0{,}35 - 2N_2 \cdot 0{,}5}{0{,}2} = 1{,}75F - 2{,}5N_2 = 713{,}375\ N$$

$$W_B = \frac{\partial W}{\partial N_2} = \sum_{i=1}^{2} \frac{N_i l_i}{E_i S_i} \cdot \frac{\partial N_i}{\partial N_2} = \frac{1}{ES} \cdot \left[(1{,}75F - 2{,}5N_2) \cdot l_1 \cdot (2{,}5) \right.$$

$$\left. + N_2 \cdot l_2 \cdot 1 \right] = 0 \quad / \cdot ES$$

$$[6{,}25 \cdot N_2 \cdot l_1 - 4{,}375F \cdot l_1 + N_2 \cdot l_2] = 0$$

$$[N_2 \cdot (6{,}25 \cdot l_1 + l_2) - 4{,}375F \cdot l_1] = 0$$

$$N_2 \cdot (6{,}25 l_1 + l_2) - 4{,}375F \cdot l_1 = 0$$

$$N_2 = \frac{4{,}375 F l_1}{(6{,}25 l_1 + l_2)} = \frac{4{,}375 \cdot 0{,}5 \cdot 2000}{(6{,}25 \cdot 0{,}5 + 0{,}8)} = \frac{4375}{3{,}925} = \underline{\underline{1114{,}65\ N}}$$

$$\sigma^{(1)} = \frac{N_1}{S_1} = \frac{713{,}375 \cdot 4}{\pi d^2} = \frac{2853{,}5}{\pi d^2} = \frac{908{,}3}{d^2}$$

$$\sigma^{(2)} = \frac{N_2}{S_2} = \frac{1114{,}65 \cdot 4}{\pi d^2} = \frac{1419{,}22}{d^2}$$

$$k_k^{(1)} = \frac{\sigma_k}{\sigma^{(1)}} \Rightarrow \sigma^{(1)} = \frac{\sigma_k}{k_k} \qquad \frac{908{,}3}{d_1^2} = 140 \cdot 10^6 \qquad d_1 = \sqrt{\frac{908{,}3}{140 \cdot 10^6}} = 0{,}0025\ m \doteq 2{,}5\ mm$$

$$k_k^{(2)} = \frac{1419{,}22}{d_2^2} = 140 \cdot 10^6 \qquad d_2 = \sqrt{\frac{1419{,}22}{140 \cdot 10^6}} = 0{,}0032\ m \doteq 3{,}2\ mm$$

$$d_{min} = \max\{d_1, d_2\} = d_2 = \underline{\underline{3{,}2\ mm}}$$