$M = 200\,Nm$

$a = 50mm$

$b = 150mm$

$c = 60mm$

$\phi d = 30mm$

$\sigma_K = 350\,Mpa$

$E = 2{,}1 \cdot 10^5\,Mpa$

$G = 80400\,MPa$, *pro* ocel

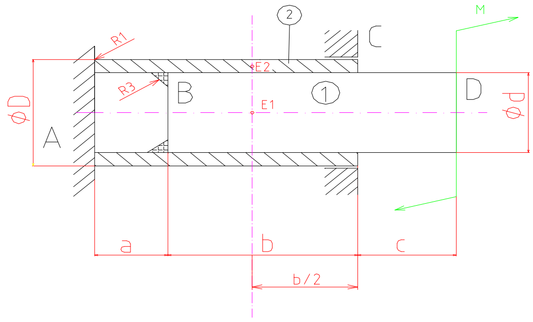
$K_K = 1{,}5$

$\varphi_{E1} = ?$

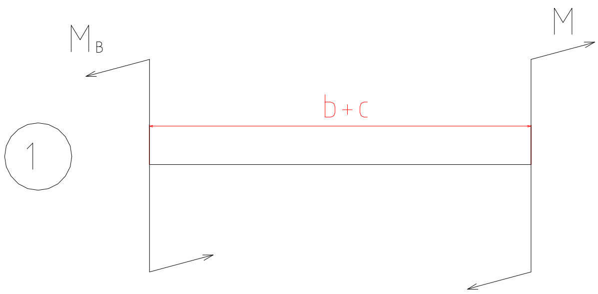
$\varphi_{E2} = ?$

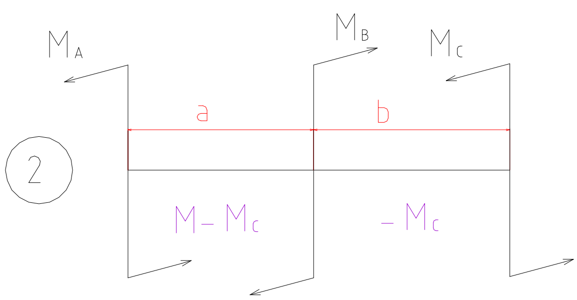
$\varphi_D = ?$

$\phi D = ?$



<u>ÚU</u>





$M_1 = 0$: $M_B - M = 0 \Rightarrow$ $\mathbf{M_B = M}$

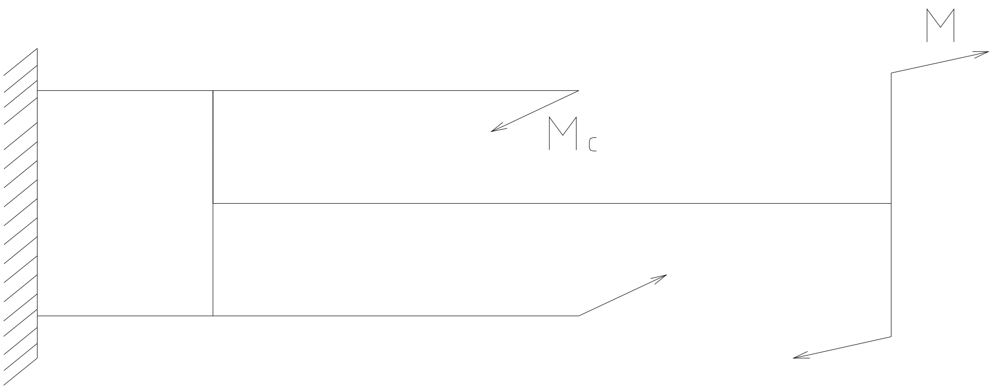
$M_2 = 0$: $M_A - M_B + M_C = 0 \Rightarrow$ $\mathbf{M_A = M - M_C}$

$\nu = 2$

$\mu = 3$

$s = 1$

ČU



$$\varphi_C = \frac{\partial W}{\partial Mc} = 0$$

$$\varphi_C = \sum_{i=1}^{3} \frac{M_{ki} \cdot l_i}{\sigma_i \cdot J_{Pi}} \cdot \frac{\partial W}{\partial Mc} = 0$$

$$\varphi_C = \sum_{i=1}^{5} \frac{M_{ki} \cdot l_i}{\sigma_i \cdot J_{Pi}} \cdot \frac{\partial Mki}{\partial Mc} = \frac{M \cdot (b+c)}{\sigma \cdot J_{P1}} \cdot 0 + \frac{(M - M_C) \cdot a}{\sigma \cdot J_{P2}} \cdot (-1) + \frac{(-M_C) \cdot b}{\sigma \cdot J_{P2}} \cdot (-1) = 0$$

$$\Rightarrow \frac{(M - M_C) \cdot a}{J_{P2}} = \frac{M_C \cdot b}{J_{P2}}$$

$$M_C = \frac{M \cdot a}{(a+b)} = 50 N / m$$

MA = M – MC = 150N/m

Výpočet průměru D

$$\frac{\tau_K}{K_K} = \frac{M_A}{J_{P2}} \cdot \frac{D}{2} = \frac{16 M_A \cdot D}{\pi \cdot (D^4 - d^4)} \Rightarrow (D^4 - d^4) = \frac{16 M_A \cdot D \cdot K_K}{\pi \cdot \tau_K}$$

$$D = \sqrt[4]{\frac{16 M_A \cdot D \cdot K_K}{\pi \cdot \tau_K}} + d = \sqrt[4]{6548{,}09 D} + d = 8{,}996 \cdot \sqrt[4]{D} + 30 = 8{,}996 \cdot \left(\sqrt[4]{D} \cong 2{,}45\right) + 30 = 52 mm$$

V průměru D je započítaný i součinitel vrubu $\alpha_A = 1{,}585$, takže průměr D podělíme α_A.

$$D^* = \frac{D}{\alpha_A} = \frac{52}{1{,}585} = 32{,}8mm$$

Výpočet $\varphi_D, \varphi_{E1}, \varphi_{E2}$

$$J_{P1} = \frac{\pi \cdot d^4}{32} = \frac{\pi \cdot 30^4}{32} = 79521{,}6mm^4$$

$$J_{P2} = \frac{\pi \cdot \left(D^4 - d^4\right)}{32} = \frac{\pi \cdot \left(32{,}8^4 - 30^4\right)}{32} = 34109mm^4$$

$$\varphi_D = \frac{M \cdot (b + c)}{\sigma \cdot J_{P1}} = \frac{200000 \cdot 210}{80400 \cdot 79521{,}6} = 6{,}57 \cdot 10^{-3} \cdot \frac{180}{\pi} = 0{,}3764^\circ$$

$$\varphi_{E1} = \frac{M_A \cdot a}{\sigma \cdot J_{P2}} + \frac{M_B \cdot \frac{b}{2}}{\sigma \cdot J_{P1}} = \frac{150000 \cdot 50}{80400 \cdot 34109} + \frac{200000 \cdot 75}{80400 \cdot 79521{,}6} = 2{,}735 \cdot 10^{-3} \cdot \frac{180}{\pi} + 2{,}346 \cdot 10^{-3} \cdot \frac{180}{\pi} = 0{,}2911^\circ$$

$$\varphi_{E2} = \frac{M_C \cdot \frac{b}{2}}{\sigma \cdot J_{P2}} = \frac{50000 \cdot 75}{80400 \cdot 34109} = 1{,}367 \cdot 10^{-3} \cdot \frac{180}{\pi} = 0{,}0783^\circ$$

$M_2 = 150 Nm$

$M_1 = 200 Nm$

$a = 150 mm$

$b = 100 mm$

$c = 300 mm$

$e = 100 mm$

$f = 150 mm$

$\sigma_K = 350 Mpa$

$E = 2{,}1 \cdot 10^5 Mpa, G = 80400 Mpa$

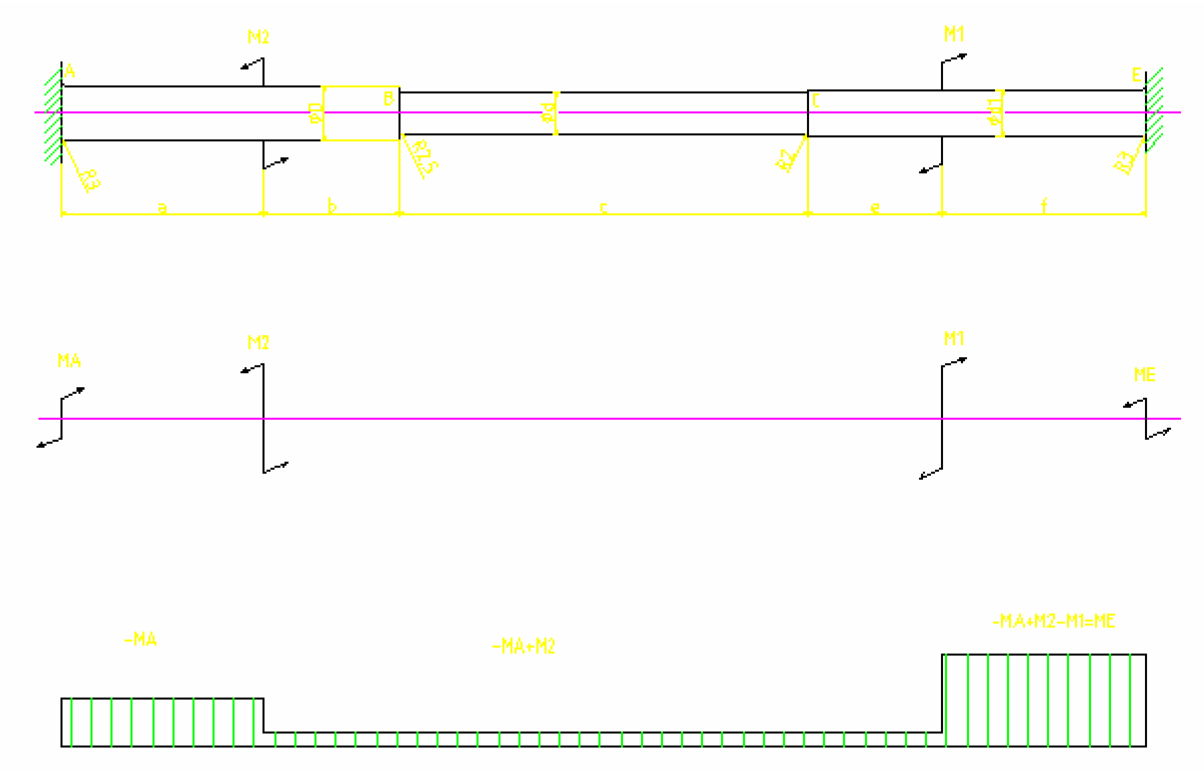
$K_K = ?$

$\varphi_c = ?$

$\varphi_d = ?$

$\varphi_e = ?$

$M_1 > M_2$



$\nu = 1$

$\mu = 2$

$s = 1$

$$\sum M := -Ma + M_2 - M_1 + Me = 0$$

$$J_{PA} = \frac{\pi \cdot D^4}{32} = \frac{\pi \cdot 40mm^4}{32} = 251327mm^4$$

$$J_{PB} = \frac{\pi \cdot d^4}{32} = \frac{\pi \cdot 30mm^4}{32} = 79251mm^4$$

$$J_{PC} = J_{PB} = 79251mm^4$$

$$J_{PE} = \frac{\pi \cdot {d_1}^4}{32} = \frac{\pi \cdot 35mm^4}{32} = 147323mm^4$$

Částečné uvolnění $\varphi_A = 0$

$$\varphi_A = \sum_{i=1}^{5} \frac{M_{ki} \cdot l_i}{G_i \cdot J_{Pi}} = \frac{-M_a \cdot a}{G \cdot J_{PA}} + \frac{(M_2 - M_a) \cdot b}{G \cdot J_{PA}} + \frac{(M_2 - M_a) \cdot c}{G \cdot J_{PB}} + \frac{(M_2 - M_a) \cdot e}{G \cdot J_{PE}} + \frac{(M_2 - M_a + M_1) \cdot f}{G \cdot J_{PE}} = 0$$

$$\Rightarrow -\frac{M_a \cdot a}{J_{PA}} - \frac{M_a \cdot b}{J_{PA}} - \frac{M_a \cdot c}{J_{PB}} - \frac{M_a \cdot e}{J_{PE}} - \frac{M_a \cdot f}{J_{PE}} = -\frac{M_2 \cdot b}{J_{PA}} - \frac{M_2 \cdot c}{J_{PB}} - \frac{M_2 \cdot e}{J_{PE}} - \frac{(M_1 + M_2) \cdot f}{J_{PE}}$$

$$M_a = \frac{\frac{M_2 \cdot b}{J_{PA}} + \frac{M_2 \cdot c}{J_{PB}} + \frac{M_2 \cdot e}{J_{PE}} + \frac{(M_1 + M_2) \cdot f}{J_{PE}}}{\left(\frac{a}{J_{PA}} + \frac{b}{J_{PA}} + \frac{c}{J_{PB}} + \frac{e}{J_{PE}} + \frac{f}{J_{PE}}\right)} =$$

$$= \frac{\frac{150000Nmm \cdot 100mm}{251327mm^4} + \frac{150000Nmm \cdot 300mm}{79251mm^4} + \frac{150000Nmm \cdot 100mm}{147323mm^4} + \frac{(200000Nmm + 150000Nmm) \cdot 150mm}{147323mm^4}}{\left(\frac{150mm}{251327mm^4} + \frac{100mm}{251327mm^4} + \frac{300mm}{79521mm^4} + \frac{100mm}{147323mm^4} + \frac{150mm}{147323mm^4}\right)}$$

$$= 167950{,}5949Nmm \doteq 168Nm$$

$$-M_a + M_2 - M_1 + M_e = 0 \Rightarrow Me = M_a - M_2 + M_1 = 168Nm - 150Nm + 200Nm = 218Nm$$

$$\tau = \frac{M_K}{J_P} \cdot R \cdot \alpha$$

$$\tau_a = \frac{M_a}{J_{PA}} \cdot R \cdot \alpha = \frac{168000 Nmm}{251327 mm^4} \cdot 20mm \cdot 1{,}75 = 23{,}39 Mpa$$

$$\tau_b = \frac{M_2 - M_a}{J_{PB}} \cdot R \cdot \alpha = \frac{150000 Nmm - 168000 Nmm}{79251 mm^4} \cdot 15mm \cdot 1{,}45 = 4{,}94 Mpa$$

$$\tau_c = \frac{M_2 - M_a}{J_{PC}} \cdot R \cdot \alpha = \frac{150000 Nmm - 168000 Nmm}{79251 mm^4} \cdot 15mm \cdot 1{,}5 = 5{,}11 Mpa$$

$$\tau_d = \frac{M_e}{J_{PE}} \cdot R \cdot \alpha = \frac{168000 Nmm}{147323 mm^4} \cdot 17{,}5mm \cdot 1{,}75 = 34{,}92 Mpa$$

$$\tau_K = \sigma_K / 2 = 350 Mpa / 2 = 175 Mpa$$

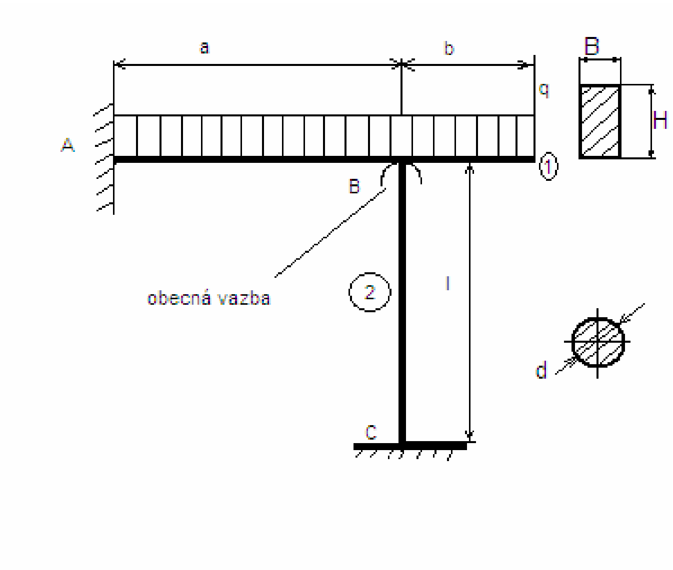
$$K_K = \frac{\tau_K}{\tau_{\max}} = \frac{175 Mpa}{34{,}92 Mpa} \doteq 5$$

$$\varphi_B = \sum_{i=1}^{3} \frac{M_{ki} \cdot l_i}{G_i \cdot J_{Pi}} = + \frac{(M_2 - M_a) \cdot c}{G \cdot J_{PB}} + \frac{(M_2 - M_a) \cdot e}{G \cdot J_{PE}} + \frac{(M_2 - M_a + M_1) \cdot f}{G \cdot J_{PE}} =$$
$$= \frac{(150000 Nmm - 168000 Nmm) \cdot 300mm}{80400 Mpa \cdot 79521 mm^4} + \frac{(150000 Nmm - 168000 Nmm) \cdot 100mm}{80400 Mpa \cdot 147323 mm^4} + \frac{(150000 Nmm - 168000 Nmm + 200000 Nmm) \cdot 150mm}{80400 Mpa \cdot 147323 mm^4} =$$
$$\doteq 1{,}307 \cdot 10^{-3} mm$$

$$\varphi_c = \sum_{i=1}^{2} \frac{M_{ki} \cdot l_i}{G_i \cdot J_{Pi}} = + \frac{(M_2 - M_a) \cdot e}{G \cdot J_{PE}} + \frac{(M_2 - M_a + M_1) \cdot f}{G \cdot J_{PE}} =$$
$$= \frac{(150000 Nmm - 168000 Nmm) \cdot 100mm}{80400 Mpa \cdot 147323 mm^4} + \frac{(150000 Nmm - 168000 Nmm + 200000 Nmm) \cdot 150mm}{80400 Mpa \cdot 147323 mm^4} =$$
$$\doteq 2{,}153 \cdot 10^{-3} mm$$

$$\varphi_A = \sum_{i=1}^{5} \frac{M_{ki} \cdot l_i}{G_i \cdot J_{Pi}} = \frac{-M_a \cdot a}{G \cdot J_{PA}} + \frac{(M_2 - M_a) \cdot b}{G \cdot J_{PA}} + \frac{(M_2 - M_a) \cdot c}{G \cdot J_{PB}} + \frac{(M_2 - M_a) \cdot e}{G \cdot J_{PE}} + \frac{(M_2 - M_a + M_1) \cdot f}{G \cdot J_{PE}} =$$
$$= \frac{-168000 Nmm \cdot 150mm}{80400 Mpa \cdot 251327 mm^4} + \frac{(150000 Nmm - 168000 Nmm) \cdot 100mm}{80400 Mpa \cdot 251327 mm^4} + \frac{(150000 Nmm - 168000 Nmm) \cdot 300mm}{80400 Mpa \cdot 79521 mm^4} + \frac{(150000 Nmm - 168000 Nmm) \cdot 100mm}{80400 Mpa \cdot 147323 mm^4} +$$
$$+ \frac{(150000 Nmm - 168000 Nmm + 200000 Nmm) \cdot 150mm}{80400 Mpa \cdot 147323 mm^4} \doteq -2{,}919 \cdot 10^{-5} mm$$

$$\varphi_E = \varphi_A + \varphi_B + \varphi_C = -2{,}919 \cdot 10^{-5} mm + 1{,}307 \cdot 10^{-3} mm + 2{,}153 \cdot 10^{-3} = 3{,}431 \cdot 10^{-3} mm$$



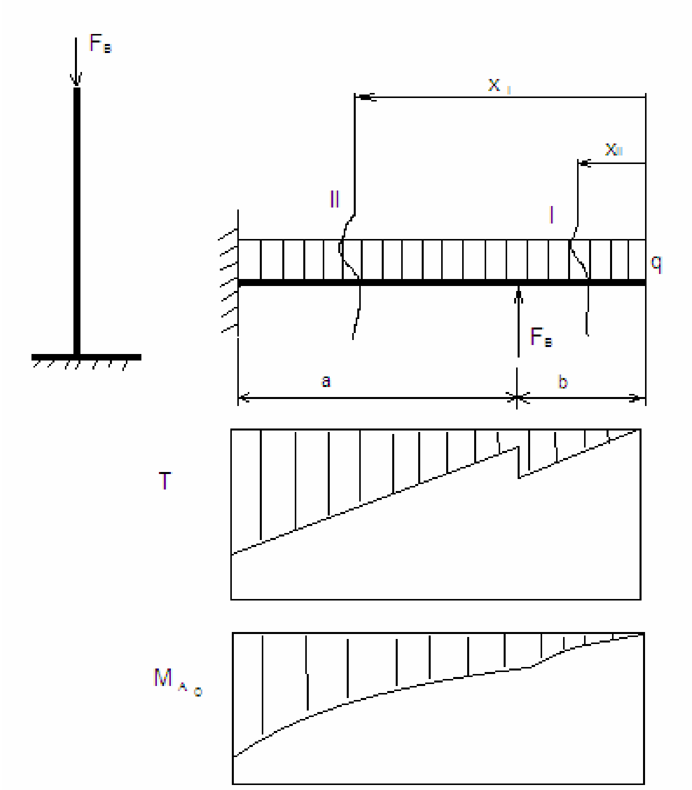
d_2=8mm
a=0,8m=800mm
b=0,2m=200mm
q=500Nm
E=2,1 . 10^5Mpa
l=0,5mm=500mm
σ_K=200Mpa
S_2=50,26mm^2
B=50mm
H=10mm

$$J_{y1} = \frac{BH^3}{12} = \frac{10 \cdot 30^3}{12} = \underline{\underline{22500mm^4}} = 2{,}25 \cdot 10^{-8}$$

$$J_{y2} = \frac{J_P}{2} = \frac{\pi \cdot D^4}{64} = \frac{\pi \cdot 8^4}{64} = \underline{\underline{201mm^4}}$$

$$S_2 = \pi \cdot R^2 = \pi \cdot 0{,}004^2 = \underline{\underline{50{,}26mm^2 = 5{,}026 \cdot 10^{-8} m^2}}$$

Č.U.



$w_B = u_B$

$$M_{oI} = \frac{-q \cdot x_1}{2} \cdot x_1 = \frac{-q \cdot x_1^2}{2} \qquad x_1 \in \langle 0, b \rangle$$

$$M_{oII} = (x_2 - b)F_B - x_2 \cdot q \cdot \frac{x_2}{2} = (x_2 - b)F_B - \frac{x_2^2 q}{2} = x_2 F_B - bF_B - x_2 - \frac{x_2^2 q}{2} \qquad x_2 \in \langle a, a+b \rangle$$

$$\frac{\partial M_{oI}}{\partial F_B} = 0 \qquad \frac{\partial M_{oII}}{\partial F_B} = (x_2 - b)$$

$$w_B = \frac{\partial W}{\partial F_B} = \frac{1}{EJ} \cdot \left[\int_0^b M_{oI} \cdot \frac{\partial M_{oI}}{\partial F_B} dx_1 + \int_b^{a+b} M_{oII} \cdot \frac{\partial M_{oII}}{\partial F_B} dx_2 \right] =$$

$$= \frac{1}{EJ} \cdot \int_b^{a+b} \left(x_2 F_B - bF_B - x_2 - \frac{x_2^{\ 2} q}{2} \right) \cdot (x_2 - b) dx_2 =$$

$$= \frac{1}{EJ} \cdot \int_b^{a+b} \left(x_2^{\ 2} F_B + bx_2 F_B - \frac{x_2^{\ 3} \cdot q}{2} - x_2 bF_B + \frac{x_2^{\ 2} bq}{2} \right) dx_2 =$$

$$= \frac{1}{EJ} \cdot \left[-\frac{x_2^{\ 4} \cdot q}{8} + \frac{x_2^{\ 3} F_B}{3} - bx_2^{\ 2} F_B + x_2 b^2 F_B + \frac{x_2^{\ 3} bq}{6} \right]_b^{b+a} \Rightarrow$$

$$\Rightarrow \frac{1}{EJ} \cdot \left[\begin{array}{l} -\frac{1^4 \cdot 500}{8} + \frac{1^3 F_B}{3} - 0{,}2 \cdot 1^2 \cdot F_B + 1 \cdot 0{,}2^2 F_B + \frac{1^3 \cdot 0{,}2 \cdot 500}{6} + \\ + \frac{1{,}6 \cdot 10^{-3} \cdot 500}{8} - \frac{0{,}2^3 F_B}{3} + 0{,}2 \cdot 0{,}2^2 \cdot F_B - 0{,}2 \cdot 0{,}2^2 F_B + \frac{0{,}2^3 \cdot 0{,}2 \cdot 500}{6} \end{array} \right] =$$

$$= \frac{1}{EJ} \cdot \left[0{,}170733333 \ \ F_B - 46{,}37 \right]$$

$$u_B = \frac{\partial W_2}{\partial F_B} = \frac{N \cdot l}{E \cdot S_2} = \frac{F_B \cdot l}{E \cdot S_2} \cdot 1$$

$$w_B = u_B$$

$$\frac{1}{EJ_{y1}} \cdot \left[0{,}170733333 F_B - 46{,}37 \right] = \frac{N \cdot l}{E \cdot S_2}$$

$$\frac{1}{EJ_{y1}} \cdot \left[0{,}170733333 F_B - 46{,}37 \right] = \frac{1}{E} (9948{,}26 F_B)$$

$$-9948{,}26 F_B + 7588148 F_B = 2060888889$$

$$\underline{\underline{F_B = 272 N}}$$

Kontrola MSP v prutu 1

$$M_A = \frac{a^2 \cdot q}{2} - F_B \cdot a + b \cdot q\left(a + \frac{b}{2}\right) = 160 - 217{,}6 + 90 = 32{,}4 N \cdot m = 32400 Nmm$$

$$\sigma_A = \frac{M_A \cdot \frac{H}{2}}{J_{y1}} = \frac{32400 \cdot 15}{22500} = 21{,}6 MPa \qquad k_k = \frac{\sigma_k}{\sigma_A} = \frac{200}{21{,}6} = 9{,}25$$ nenastane MSP

Kontrola MSP v prutu 2

$$\sigma = \frac{F_B}{S_2} = \frac{271}{201} = \underline{\underline{1{,}34 MPa}} << \sigma_{dov}$$ nenastane MSP

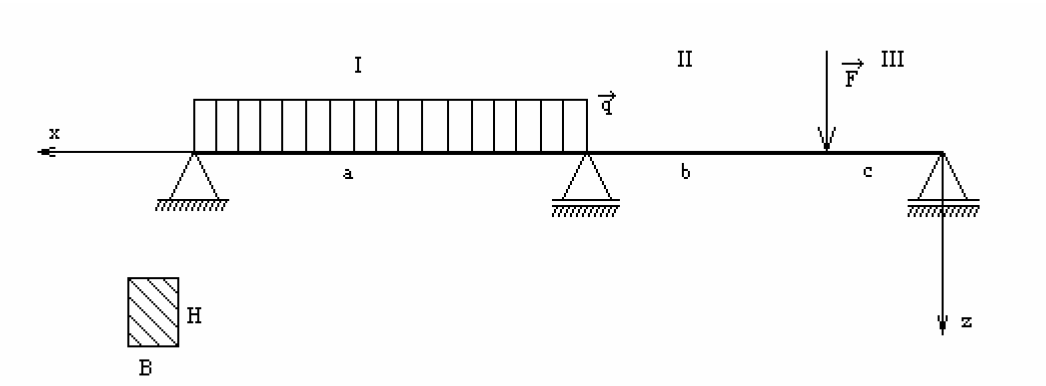
Kontrola MSVS v prutu 2

$$\alpha = \frac{\pi}{2} \qquad F_v = \frac{\alpha^2 \cdot E \cdot J_{y2}}{l^2} = \frac{\pi^2 \cdot E \cdot J_{y2}}{4 \cdot l^2} = \frac{\pi^2 \cdot 2{,}1 \cdot 10^5 \cdot 201}{4 \cdot 500^2} = \underline{\underline{417 N}}$$

$$\Rightarrow F_B < F_v \quad \Rightarrow k_k = \frac{417}{272} = 1{,}5 \Rightarrow$$ nenastane Mezní stav vzpěrné stability

Diferenciální ohyb

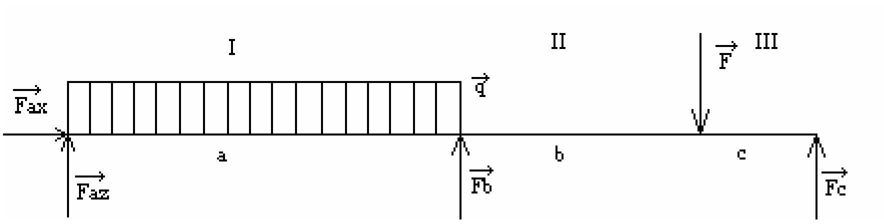
Zadání:



a=0,8m, b=0,2m, c=0,1m, B=0,01m, H=0,02m, F=130N, q=500Nm, σ_k=320Mpa, E=2,1.10^{5}MPa

$\varphi(A)$=?, $\varphi(B)$=?, $\varphi(C)$=?, w_{max}=?

Úplné uvolnění + rovnice rovnováhy:



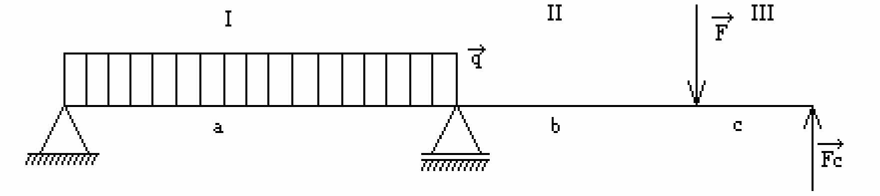
$$NP = \{F_{Ax}, F_{Az}, F_B, F_C\}$$

$s = \mu - \nu = 4 - 3 = 1 \quad \rightarrow$ úloha je 1x staticky neurčitá

Rovnice rovnováhy:

$$\sum F_x : F_{Ax} = 0$$
$$\sum F_y : -F_{Ay} - F_B - F_c + F + qa = 0$$
$$\sum M_o : -qa\left(\frac{a}{2} + b + c\right) + F_B(b + c) - F.c + F_C.0 = 0$$

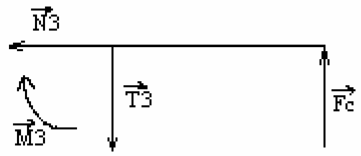
Částečné uvolnění + Castiglianova věta:



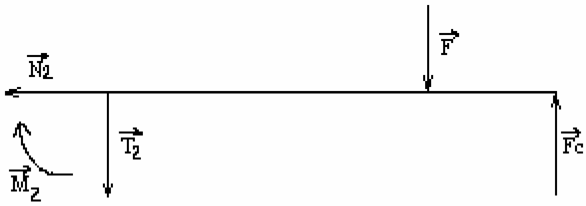
Castiglianova věta:

$$w_C = 0 = \frac{\partial W}{\partial F_C} = \sum \int_\gamma \frac{M_{oi}}{EJ_y} \frac{\partial M_{oi}}{\partial F_C} dx_i$$

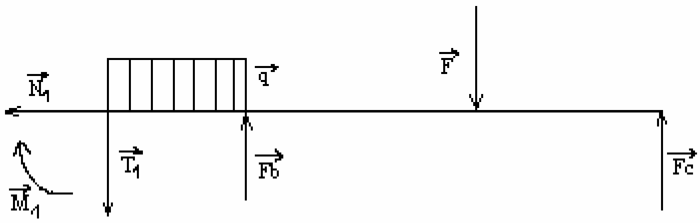
<u>Určení momentů:</u>



$$M_3 = F_C x_{III}$$



$$M_2 = F_C x_{II} - F(x_{II} - c)$$



$$M_1 = F_C x_I - F(x_{II} - c) + F_B(x_I - b - c) - qa\left(x_I - \frac{a}{2} - b - c\right)$$

$$\frac{\partial M_I}{\partial F_C} = x_I \qquad \frac{\partial M_{II}}{\partial F_C} = x_{II} \qquad \frac{\partial M_{III}}{\partial F_C} = x_{III}$$

$$w_C = \left(\int_{b+c}^{a+b+c} \frac{M_{oI}}{EJ_y} x_I dx_I + \int_{c}^{b+c} \frac{M_{oII}}{EJ_y} x_{II} dx_{II} + \int_{0}^{c} \frac{M_{oIII}}{EJ_y} x_{III} dx_{III} \right) =$$

$$= \int_{0,3}^{1,1} \frac{-qa\left(x_I - b - c - \frac{a}{2}\right) + F_B(x_I - b - c) - F(x_I - c) + F_C x_I}{EJ_y} x_I dx_I +$$

$$+ \int_{0,1}^{0,3} \frac{F_C x_{II} - F x_{II} + Fc}{EJ_y} x_{II} dx_{II} + \int_{0}^{0,1} \frac{F_C x_{III}}{EJ_y} x_{III} dx_{III} =$$

$$= \frac{1}{EJ_y} \left[\frac{-qax_I^3}{3} + \frac{qabx_I^2}{2} + \frac{qacx_I^2}{2} + \frac{qa^2 x_I^2}{2.2} + \frac{F_B x_I^3}{3} - \frac{F_B b x_I^2}{2} - \frac{F_B c x_I^2}{2} - \frac{F x_I^3}{3} + \frac{F c x_I^2}{2} + \frac{F_C x_I^3}{3} \right]_{0,3}^{1,1} +$$

$$+ \frac{1}{EJ_y} \left[\frac{F_C x_{II}^3}{3} - \frac{F x_{II}^3}{3} + \frac{F c x_{II}^2}{2} \right]_{0,1}^{0,3} + \frac{1}{EJ_y} \left[\frac{F x_{III}^3}{3} \right]_{0}^{0,1} =$$

$$= \frac{1}{EJ_y} \left[\left(\frac{-51,2}{3} + \frac{0,8}{3} F_B - \frac{1,136}{3} F + \frac{1,304}{3} F_C \right) + \left(\frac{0,026}{3} F_C - \frac{0,014}{3} F \right) + \left(\frac{0,001}{3} F \right) \right]$$

$$\frac{1}{EJ_y} \left[\left(\frac{-51,2}{3} + \frac{0,8}{3} F_B - \frac{1,136}{3} F + \frac{1,304}{3} F_C \right) + \left(\frac{0,026}{3} F_C - \frac{0,014}{3} F \right) + \left(\frac{0,001}{3} F \right) \right] = 0$$

$$\underline{\underline{-51,2 + 0,8 F_B - 1,149 F + 1,33 F_C = 0}}$$

Castiglianova věta + rovnice rovnováhy (soustava 3 rovnic o 3 neznámých):

Ze zadání: F=130N

1.) $-51,2 + 0,8F_B - 1,149F + 1,33F_C = 0$

$2400 - F_B + F - F_C - F_{Ay} = 0$

3.) $-280 + 0,3F_B - 0,1F = 0$

4.) $F_{Ax} = 0$

ze 3. rovnice: $F_B = \frac{280 + 0,1.130}{0,3} = \underline{\underline{976,66\text{N}}}$

z 1. rovnice: $F_C = \frac{1,149.130 - 0,8.976,66 + 51,2}{1,33} = \underline{\underline{-436,66\text{N}}}$

ze 2. rovnice: $F_B = 400 - 976,66 + 130 + 436,66 = \underline{\underline{-10\text{N}}}$

Rovnice průhybových čar:

$$w_I'' = -\frac{M_{oI}}{EJ_y} = \frac{1}{EJ_y}\left(-F(x_I - b - c) + F(x_I - c) - F_C x_I + qa\left(x_I - b - c - \frac{a}{2}\right)\right)$$

$$w_I' = \frac{1}{EJ_y}\left(\frac{-F_B x_I^2}{2} + F_B b x_I + F_B c x_I + \frac{F x_I^2}{2} - F c x_I - \frac{F_C x_I^2}{2} + \frac{qa x_I^2}{2} - qab x_I - qac x_I - \frac{qa^2 x_I^2}{2} + c_1\right)$$

$$w_I = \frac{1}{EJ_y}\left(\frac{-F_B x_I^3}{6} + \frac{F_B b x_I^2}{2} + \frac{F_B c x_I^2}{2} + \frac{F x_I^3}{6} - \frac{F c x_I^2}{2} - \frac{F_C x_I^3}{6} + \frac{qa x_I^3}{6} - \frac{qab x_I^2}{2} - \frac{qac x_I^2}{2} - \frac{qa^2 x_I^2}{2.2} + c_1 x_I + c_2\right)$$

$$w_{II}'' = -\frac{M_{oII}}{EJ_y} = \frac{1}{EJ_y}\left(-F_C x_{II} + F x_{II} - Fc\right)$$

$$w_{II}' = \frac{1}{EJ_y}\left(-\frac{F_C x_{II}^2}{2} + \frac{F x_{II}^2}{2} - F c x_{II} + c_3\right)$$

$$w_{II} = \frac{1}{EJ_y}\left(\frac{-F_C x_{II}^3}{6} + \frac{F x_{II}^3}{6} - \frac{F c x_{II}^2}{2} + c_3 x_{II} + c_4\right)$$

$$w_{III}'' = -\frac{M_{oIII}}{EJ_y} = \frac{1}{EJ_y}\left(-F_C x_{III}\right)$$

$$w_{III}' = \frac{1}{EJ_y}\left(-\frac{F_C x_{III}^2}{2} + c_5\right)$$

$$w_{III} = \frac{1}{EJ_y}\left(\frac{-F_C x_{II}^3}{6} + c_5 x_{III} + c_6\right)$$

Okrajové podmínky:

1. $w_I = (x_I = a + b + c) = 0$

2. $w_{II} = (x_{II} = b + c) = 0$

3. $w_{III} = (x_{III} = 0) = 0$

4. $w_{III} = (x_{III} = c) = w_{II}(x_{II} = c)$

5. $w_{II}' = (x_{II} = c) = w_{III}'(x_{III} = c)$

6. $w_I = (x_I = b + c) = 0$

Výpočet integračních konstant:

Dosazení do okrajových podmínek:

1.) $-216{,}66 + 118{,}176 + 59{,}09 + 28{,}84 - 7{,}865 + 96{,}866 + 88{,}73 - 48{,}4 - 24{,}2 - 96{,}8 + 1{,}1c_1 + c_2 = 0$

$\underline{-2{,}223 + 1{,}1c_1 + c_2 = 0}$

2.) $1{,}965+1{,}95-0{,}585+0{,}3c_3+c_4=0$
$\underline{3{,}33+0{,}3c_3+c_4=0}$

3.) $\underline{c_6=0}$

4.) $0{,}0728+0{,}1c_5=0{,}0728+0{,}2166-0{,}065+0{,}1c_3+c_4$
$\underline{-0{,}1516+0{,}1c_5-0{,}1c_3-c_4=0}$

5.) $2{,}183+0{,}65-1{,}35+c_3=2{,}183+c_5$
$\underline{-0{,}7+c_3+c_5=0}$

6.) $-4{,}39+8{,}79+4{,}39+0{,}585-0{,}585-1{,}96+1{,}8-3{,}6-1{,}8-7{,}2+0{,}3c_1+c_2=0$
$\underline{-3{,}97+0{,}3c_1+c_2=0}$

z rovnice 1.) → $\underline{c_2=2{,}223-1{,}1c_1}$

c_2 dosadím do rovnice 6.) $-3{,}97+0{,}3c_1+2{,}223-1{,}1c_1=0\rightarrow c_1=\dfrac{1{,}747}{-0{,}8}=\underline{\underline{-2{,}184}}$

dosadím c_1 do 1.) → $c_2=2{,}223-1{,}1(-2{,}184)=\underline{\underline{0{,}179}}$

z 5.) vyjádřím $c_3=c_5+0{,}7$ a dosadím do 2.) a 4.) – dostanu 2 rovnice o dvou neznámých:
$-0{,}1516+0{,}1c_5-0{,}1(c_5+0{,}7)-c_4=0$
$3{,}33+0{,}3(c_5+0{,}7)+c_4=0$
$\rightarrow c_5=\dfrac{-3{,}33-0{,}21+0{,}2216}{0{,}3}=\underline{\underline{-11{,}06}}$
$\rightarrow c_4=\underline{\underline{-0{,}2216}}$

dosadím c_5 do 5.) $\rightarrow c_3=\underline{\underline{-10{,}36}}$

<u>Průhyby a natočení:</u>

$$w_{\max}^1=w_I\left(x_I=\frac{a}{2}+b+c\right)=\frac{1}{EJ_y}\left(-55{,}8+47{,}856+23{,}93+7{,}43-3{,}185+24{,}96+22{,}866-19{,}6-\right.$$
$$\left.-9{,}8-39{,}2-1{,}5288+0{,}179\right)=\frac{1}{1260}(-1{,}8928)=-0{,}001502mm$$
$$w_{\max}^2=w_{III}(x_{III}=c)=\frac{1}{EJ_y}(0{,}07277-1{,}106)=\frac{1}{1260}(-1{,}03323)=-0{,}00082mm$$

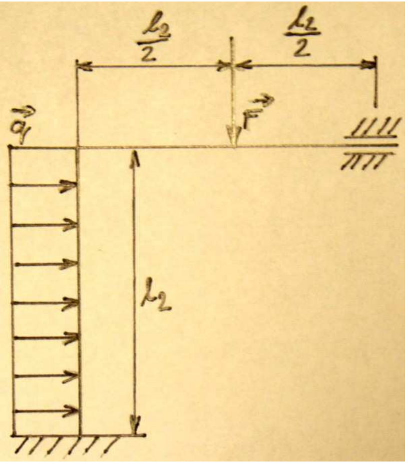
$$w_{\max}=\max\{w_{\max}^1,w_{\max}^2\}=\underline{\underline{-0{,}0015102mm}}$$

$$\varphi_A = w_I^{'}(x_I = a+b+c) = \frac{1}{EJ_y}(-590{,}88 + 214{,}86 + 107{,}4 + 78{,}65 - 14{,}3 + 264{,}2 + 242 - 88 -$$

$$-44 - 176 - 2{,}184) = \frac{-8{,}254}{1260} = \underline{\underline{-0{,}00655 rad}}$$

$$\varphi_B = w_{II}^{'}(x_{II} = b+c) = \frac{1}{EJ_y}(19{,}65 + 5{,}85 - 3{,}9 - 10{,}36) = \frac{11{,}24}{1260} = \underline{\underline{0{,}00892 rad}}$$

$$\varphi_C = w_{III}^{'}(x_{III} = 0) = \frac{1}{EJ_y}(-11{,}06) = \frac{-11{,}06}{1260} = \underline{\underline{-0{,}00877 rad}}$$



q = $200 Nm^{-1}$

F = 1000 N

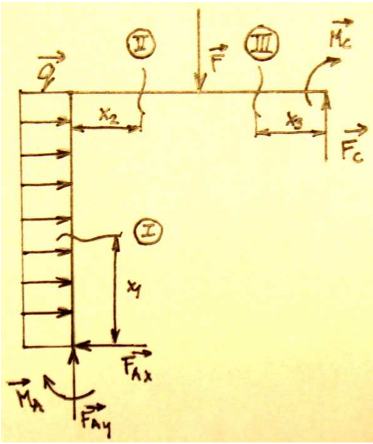
$l_2 = 400mm$

$E = 2{,}1 * 10^5 MPa$

$\sigma_k = 400MPa$

$\phi d = 30mm$

1.ÚU



$\mu = 5$
$\upsilon = 3$ => s = 2 – úloha je 2x staticky neurčitá

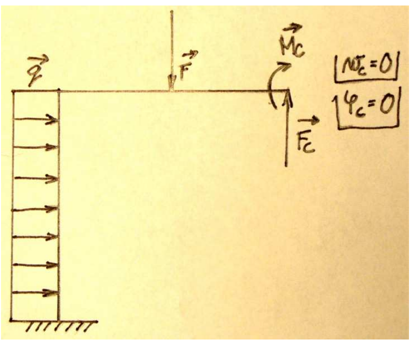
$$\sum F_x = 0: \quad -F_{Ax} + F_q = 0 \Rightarrow F_q = F_{Ax} = 200 * 0{,}4 = 80N$$

$$\sum F_y = 0: \quad F_{Ay} - F + F_C = 0 \Rightarrow F_{Ay} = 1000 - F_C$$

$$\sum M_B = 0: \quad M_A + F_{Ay} * \frac{l_2}{2} + M_C - F_C * \frac{l_2}{2} + F_{Ax} * l_1 - F_q * \frac{l_1}{2} = 0 \Rightarrow$$

$$\Rightarrow M_A = F_q * \frac{l_1}{2} - F_{Ay} * \frac{l_2}{2} - M_C + F_C * \frac{l_2}{2} - F_{Ax} * l_1$$

2.ČU



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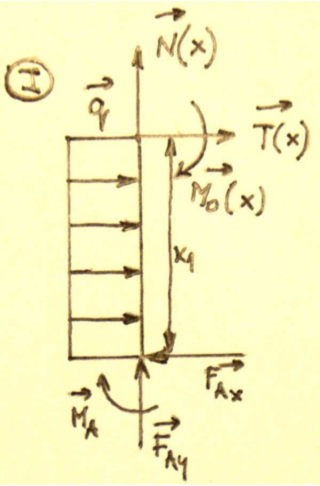
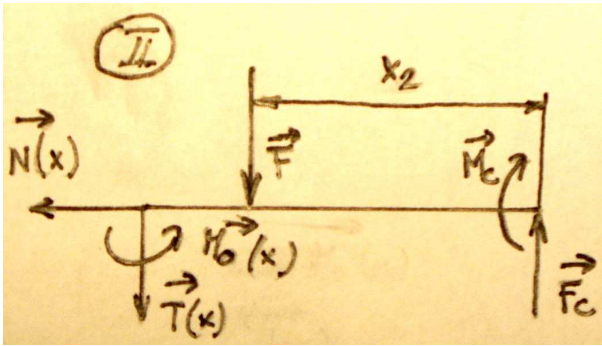
3.VVÚ

řez I : $x_1 \in \langle 0; l_1 \rangle$



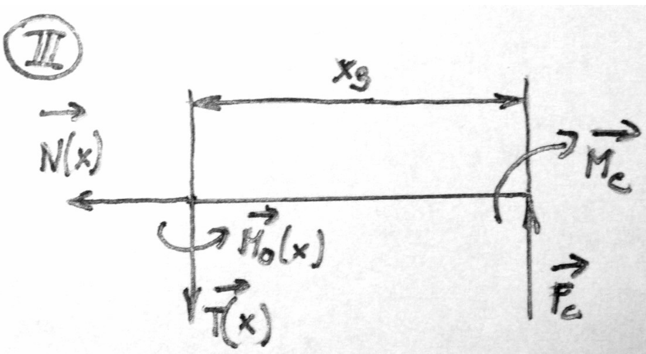
$$N^I(x) = -F_{Ay}$$
$$T^I(x) = F_{Ax} - q * x_1$$
$$M_o^I(x) = -M_A - F_{Ax} * x + q * x * \frac{x}{2}$$

řez II : $x_2 \in \langle \frac{l_2}{2}; l_2 \rangle$



$$N^{II}(x) = 0$$
$$T^{II}(x) = F_C - F$$
$$M_o^{II}(x) = M_C - F_C * x + F * (x - \frac{l_2}{2})$$

řez III : $x_3 \in \langle 0; \frac{l_2}{2} \rangle$



$$N^{III}(x) = 0$$
$$T^{III}(x) = F_C$$
$$M_o^{III}(x) = M_C - F_C * x$$

.

4.Výpočet Fc a Mc

úloha je 2xSN a máme 2 podmínky, z kterých určíme 2 neznáme
Pozn.: $l_1 = l_2$; $x_1 = x_2 = x_3 = x$

$$w_C = 0 = \frac{\partial W}{\partial F_C} = \int_0^{l_2} \frac{M_o}{E*J} * \frac{\partial M_o}{\partial F_C} * ds =$$

$$= -\frac{1}{E*J} * \left[\int_0^{l_1} (-F_q * \frac{l_1}{2} + F_{Ay} * \frac{l_2}{2} + M_C - F_C * \frac{l_2}{2} + F_{Ax} * l_1 - F_{Ax} * x + q * x * \frac{x}{2}) * l_2 dx + \right.$$

$$\left. + \int_{\frac{l_2}{2}}^{l_2} (M_C - F_C * x + F * (x - \frac{l_2}{2}) * (x - l_2) dx + \int_0^{\frac{l_2}{2}} (M_C - F_C * x) * (x - l_2) * dx \right] = 0 \Rightarrow$$

$$\Rightarrow -57{,}504 + 0{,}100632 F_C - 0{,}21564 M_C = 0$$

$$\varphi_C = 0 = \frac{\partial W}{\partial M_C} = \int_0^{l_2} \frac{M_C}{E*J} * \frac{\partial M_o}{\partial M_C} ds =$$

$$= \frac{1}{E*J} * \left[\int_0^{l_1} (-F_q * \frac{l_1}{2} + F_{Ay} * \frac{l_2}{2} + M_C - F_C * \frac{l_2}{2} + F_{Ax} * l_1 - F_{Ax} * x + q * x * \frac{x}{2}) * 1 dx + \right.$$

$$\left. + \int_{\frac{l_2}{2}}^{l_2} (M_C - F_C * x + F * (x - \frac{l_2}{2}) * 1 dx + \int_0^{\frac{l_2}{2}} (M_C - F_C * x) * 1 dx \right] = 0 \Rightarrow$$

$$\Rightarrow 310{,}5216 - 0{,}57504 F_C + 1{,}4376 M_C = 0$$

Vyjádření Fc a Mc

$$F_C = 1084{,}4 N$$
$$M_C = 239{,}38 Nm$$

5. Výpočet Fay a Ma

$$F_{Ay} = 1000 - F_C = -84{,}4 N$$

$$M_A = F_q * \frac{l_1}{2} - F_{Ay} * \frac{l_2}{2} - M_C + F_C * \frac{l_2}{2} - F_{Ax} * l_1 = -55{,}38 Nm$$

.

6. Výpočet momentů z řezů

Z řezu I :

$$M_o^I(x) = -M_A - F_{Ax} * x + q * x * \frac{x}{2}$$

$$M_o^I(0) = 55{,}38Nm$$
$$M_o^I(0{,}4) = 39{,}38Nm$$

Z řezu II :

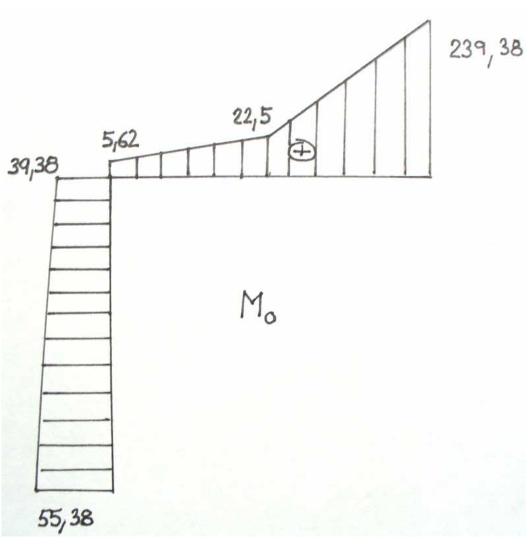
$$M_o^{II}(x) = M_C - F_C * x + F * (x - \frac{l_2}{2})$$

$$M_o^{II}(0{,}2) = 22{,}5Nm$$
$$M_o^{II}(0{,}4) = 5{,}62Nm$$

Z řezu III :

$$M_o^{II}(x) = M_C - F_C * x$$

$$M_o^{II}(0) = 239{,}38Nm$$
$$M_o^{II}(0{,}2) = 22{,}5Nm$$



7. Určení max.momentu soustavy

$$M_{o\max} = M_o^{II} = 239{,}38Nm$$

8. Výpočet bezpečnosti z max.momentu soustavy

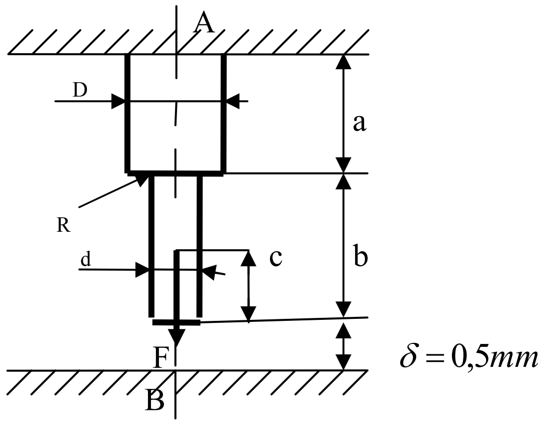
max.napětí

$$\sigma_{\max} = \frac{Mo_{\max}}{Wo} = \frac{32 * Mo_{\max}}{\pi * d^3} = 90307579{,}9Pa = 90{,}31MPa$$

bezpečnost

$$k_k = \frac{\sigma_k}{\sigma_{\max}} = \frac{400}{90{,}31} = 4{,}43$$

Zadání:



Pro jednotlivé možnosti ***a, b, c*** vypočtěte bezpečnost….
a) vazba B je nefunkční
b) $\delta = 0{,}5$
c) $\delta = 0$

Hodnoty:

D = Ø50 mm
d = Ø20mm
c = 75mm
a = 100mm
b = 150mm
R_1 = 3mm
F = 30 000 N
E = $2{,}1e^5$ MPa
$\sigma_k = R_e$ = 300MPa
K_K = ?

a) Vazba B je nefunkční (nedotkne se)

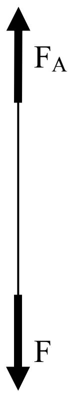
R_e= 300 MPa
D = 50 mm
d = 20 mm
R= 3 mm

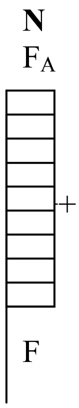
SOUČINITEL KONCENTRACE α v přechodu osazeného dříku namáhaného tahem najdeme ve skriptech str. 182.

$\frac{D}{d} = 2{,}5$ $\frac{R}{d} = 0{,}15$ a z grafu vyčteme hodnotu α = 1,75

Ú.U.:

F_y: F_A=F





Soustava sil na jedné nositelce: μ = 1; υ = 1; s = 0 Těleso je v SR

$$\sigma_{nom} = \frac{N}{S} = \frac{4F}{\pi * d^2} = \frac{4 * 30000}{\pi * 20^2} = 95{,}5 MPa$$

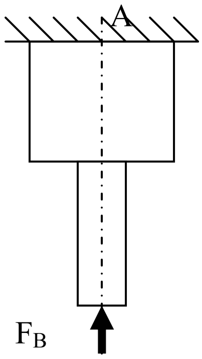
$$\sigma_{max} = \sigma_{nom} * \alpha = 95{,}5 * 1{,}75 = 167\ MPa$$

$$k_k = \frac{\sigma_k}{\sigma_{max}} = \frac{300}{167} = 1{,}79$$

c) $\delta = 0$

Soustava sil na jedné nositelce: $\mu = 2$; $\upsilon = 1$; s = 1 Chybí nám jedna rovnice. Tu získáme z deformační podmínky a zároveň uvolníme na SR.

Č.U.:



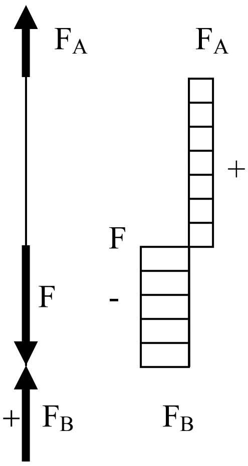
deformační podmínka: $u_B = \dfrac{\partial W}{\partial F_B} = \sum_{i=1}^{3} \dfrac{N_i l_i}{ES_i} * \dfrac{\partial N_i}{\partial F_B} = 0$

XЄ(0;c) XЄ (c;b) XЄ(b;a)

$N_1 = -\ F_B$ $N_2 = \mathrm{F} - F_B$ $N_2 = \mathrm{F} - F_B$

$\dfrac{\partial N_1}{\partial F_B} = -\ 1$ $\dfrac{\partial N_2}{\partial F_B} = -\ 1$ $\dfrac{\partial N_2}{\partial F_B} = -\ 1$

Ú.U.:



$$0 = \frac{\partial W}{\partial F_B} = \sum_{i=1}^{3} \frac{N_i l_i}{ES_i} * \frac{\partial N_i}{\partial F_B} = \frac{4*(-F_B)*c}{E*\pi*d^2}*(-1)+$$

$$+\frac{4*(F-F_B)*(b-c)}{E*\pi*d^2}*(-1)+\frac{4*(F-F_B)*a}{E*\pi*D^2}*(-1)$$

$$0 = \frac{4*(-F_B)*75}{E*\pi*20^2}*(-1\ )+\frac{4*(30000-F_B)*(150-75)}{E*\pi*20^2}*(-1\ )+$$

$$+\frac{4*(30000-F_B)*100}{E*\pi*50^2}*(-1\ )$$

$$F_B = -26566{,}7N$$

$$F_A = F_B - F = 26566{,}7 - 30000 = -3433{,}3N$$

$$\sigma_{nom} = \frac{N}{S} = \frac{4(F-F_B)}{\pi d^2} = \frac{4*(30000-26566{,}7)}{\pi*20^2} = 10{,}9MPa$$

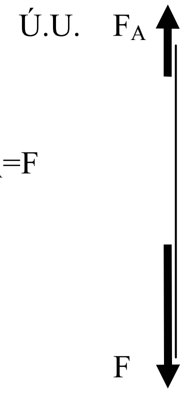
$$\sigma_{\max} = \sigma_{nom} * \alpha = 10{,}9*1{,}75 = 19{,}1MPa$$

$$k_k = \frac{\sigma_k}{\sigma_{\max}} = \frac{300}{19{,}1} = 15{,}7$$

Bezpečnost je o tolik vyšší, protože deformace je omezena působící silou F_B tím pádem je možnost porušení snížena.

b) Předpoklad – Nedojde k vymezení vůle tj. vazba B zůstane nefunkční. Vypočítáme prodloužení, kontrola vymezení vůle a případné přepočítání a kontrola na vzpěr.

Deformační podmínka : $u \ \geq \frac{\partial W}{\partial F} \leq 0{,}5$



Prodloužení

$$u = \frac{\partial W}{\partial F} = \sum_{i=1}^{2} \frac{N_i l_i}{ES_i} * \frac{\partial N_i}{\partial F}$$

F_y: $F_A = F$

Derivace:

$$\frac{\partial N_1}{\partial F} = 1 \qquad \frac{\partial N_2}{\partial F} = 1$$

Celkové prodloužení:

$$u = \frac{\partial W_{(F_B,F)}}{\partial F_B} = \sum_{i=1}^{2} \frac{N_i l_i}{ES_i} * \frac{\partial N_i}{\partial F} = \frac{4 * F * a}{E * \pi * D^2} * 1 + \frac{4 * F * (b-c)}{E * \pi * d^2} * 1 = \frac{4 * 30000 * 100}{E * \pi * 50^2} * 1 +$$
$$+ \frac{4 * 30000 * 75}{E * \pi * 20^2} * 1 = 0{,}041 mm$$

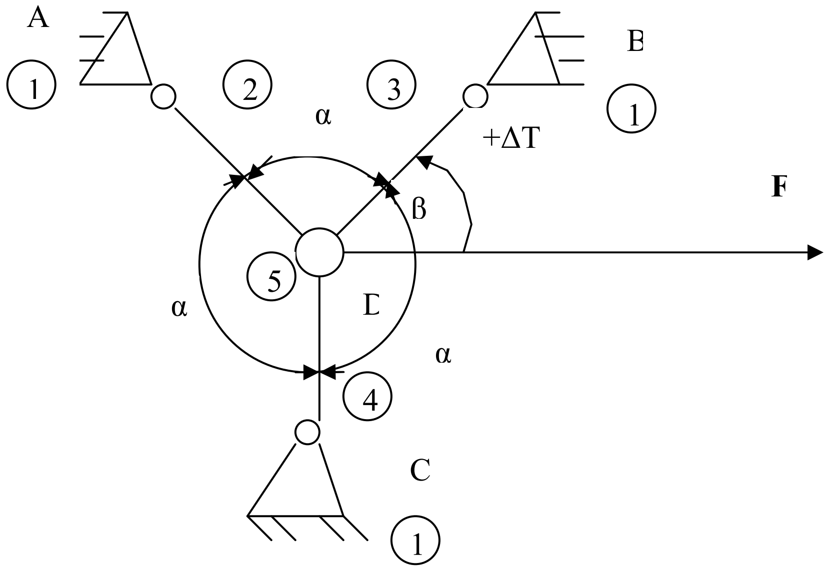
Nedojde k vymezení vůle.

Soustava sil na jedné nositelce: $\mu = 1$; $\upsilon = 1$; s = 0 Těleso je v SR

$$\sigma_{nom} = \frac{N}{S} = \frac{4F}{\pi * d^2} = \frac{4 * 30000}{\pi * 20^2} = 95{,}5 MPa$$

$$\sigma_{max} = \sigma_{nom} * \alpha = 95{,}5 * 1{,}75 = 167 \ MPa$$

$$k_k = \frac{\sigma_k}{\sigma_{max}} = \frac{300}{167} = 1{,}79$$



Mat.: ocel 13 141.5

k_k=?

ød=30mm

l_2=800mm=0,8m

l_3=900mm=0,9m

l_4=1000mm=1m

α=120°

β=30°

F=10^4 N

ΔT=50°C

koef. Teplotní roztažnosti $\alpha=1,2*10^{-5}$

$E=2,1*10^{11}$

σ_k=490 MPa

<u>Rozbor:</u>

Tělesa mají charakter prutů

a) kinematický:

A, B, C – r. k. d. – $\xi_i=2$

D – vícenásobná r. k. d. - $\xi_i=6$ (těl. 5 – degenerované těleso)

$$(i-\eta)=(n-1)i_v-\sum\xi_i-a$$

$$(i-\eta)=(5-1)*3-(3*2+6)-1=-1$$

i=0, η=-1 – nepohyblivá soustava s 1 omezeným deformačním parametrem

b) statický:

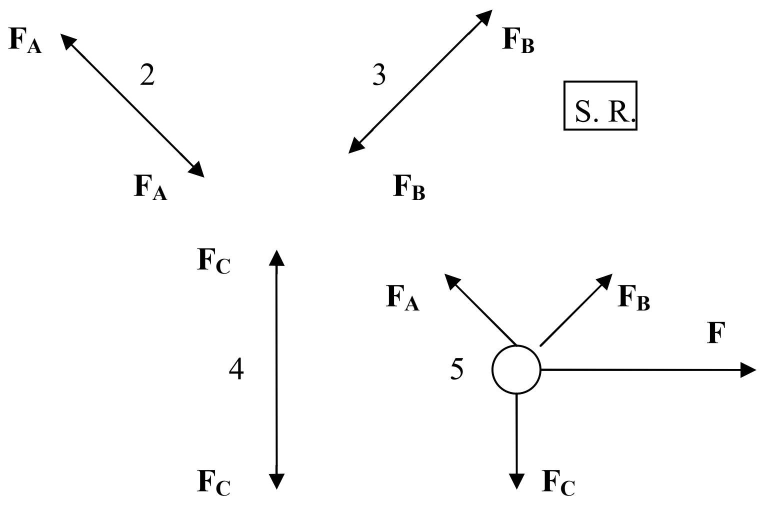
NP={F_A, F_B, F_C} μ=3

$\pi\upsilon^5$ – rovinná se společným působištěm – υ=2

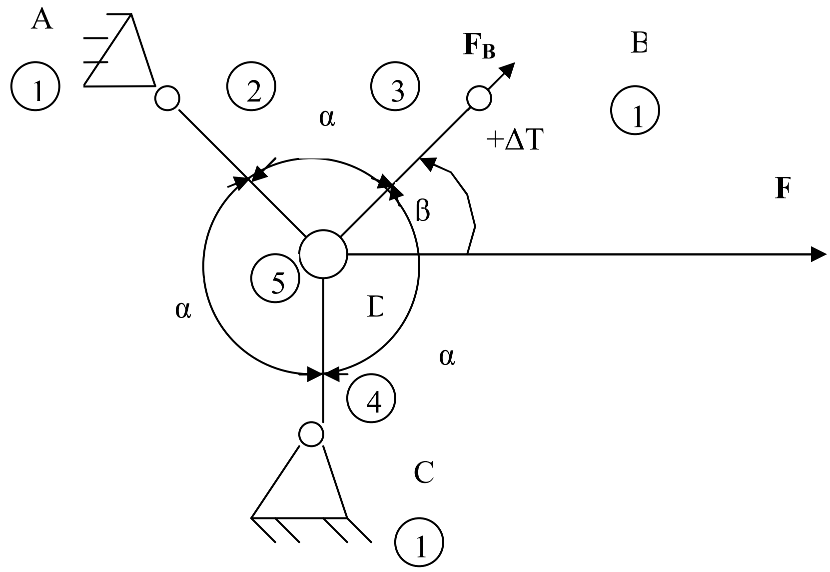
s=μ-υ=3-2=1 – 1x staticky neurčité

<u>Řešení:</u>

Úplné uvolnění:



Částečné uvolnění:



$$u_B = 0 = u_{B\Delta T} + \frac{\partial W}{\partial F_B}$$

VVÚ:

$N_2 = F_A$

$N_3 = F_B$

$N_4 = F_C$

$$F_x : F + F_B \cos\beta - F_A \cos\beta = 0$$
$$F_y : F_B \sin\beta + F_A \sin\beta - F_C = 0$$

$$N_2 = F_A = \frac{F}{\cos\beta} + F_B$$

$$N_4 = F_C = (F_A + F_B)\sin\beta = (\frac{F}{\cos\beta} + F_B + F_B)\sin\beta = F\tan\beta + 2F_B \sin\beta$$

Deformační podmínka:

$$u_B = u_{B\Delta T} + \frac{\partial W}{\partial F_B} = \alpha.l_3.\Delta T + \sum_2^4 \frac{N_i.l_i}{E.S_i}.\frac{\partial N_i}{\partial F_B} = 0$$

$$\frac{\partial N_2}{\partial F_B} = 1$$

$$\frac{\partial N_3}{\partial F_B} = 1$$

$$\frac{\partial N_4}{\partial F_B} = \sin\beta$$

$$\alpha.l_3.\Delta T + \sum_2^4 \frac{N_i.l_i}{E.S_i}.\frac{\partial N_i}{\partial F_B} = \alpha.l_3.\Delta T + \frac{N_2.l_2}{E.S_2}.\frac{\partial N_2}{\partial F_B} + \frac{N_3.l_3}{E.S_3}.\frac{\partial N_3}{\partial F_B} + \frac{N_4.l_4}{E.S_4}.\frac{\partial N_4}{\partial F_B} =$$

$$= 1{,}2.10^{-5}.0{,}9.50 + \frac{1}{2{,}1.10^{11}.7{,}07.10^{-4}}.[(\frac{10^4}{\cos 30°} + F_B).0{,}8.1 + F_B.0{,}9.1 + (F\tan\beta + 2F_B\sin\beta).1.\sin\beta] =$$

$$= 5{,}4.10^{-4} + 6{,}735.10^{-9}.(9237{,}604 + 0{,}8.F_B + 0{,}9.F_B + 2886{,}751 + 0{,}5.F_B) = 0$$

$$F_B = \frac{\frac{-5{,}4.10^{-4}}{6{,}735.10^{-9}} - (9237{,}604 + 2886{,}751)}{0{,}8 + 0{,}9 + 0{,}5} = -41955{,}695N$$

$$F_A = \frac{10^4}{\cos 30°} - 41955{,}695 = -30408{,}690N$$

$$F_C = 10^4 \tan 30° + 2.(-41955{,}695).\sin 30° = -36182{,}192N$$

$$\sigma_2 = \frac{F_A}{S} = -43MPa$$

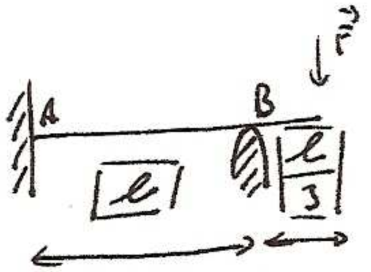
$$\sigma_3 = \frac{F_B}{S} = -59{,}3MPa$$

$$\sigma_4 = \frac{F_C}{S} = -51{,}2MPa$$

$$k_k = \frac{\sigma_k}{|\sigma_{\max}|} = \frac{490}{59{,}3} = 8{,}26$$

Př.

Určete rozměry příčného průřezu při ohybovém zatěžování tělesa silou $\vec{F}$



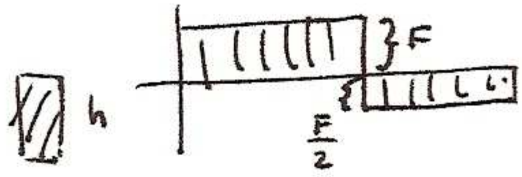
11500

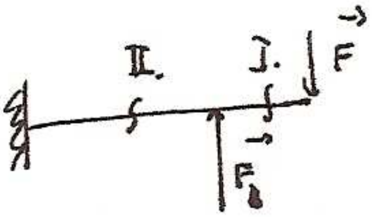
$l = 100$

$F = 5\ kN$

$k_K = 2$

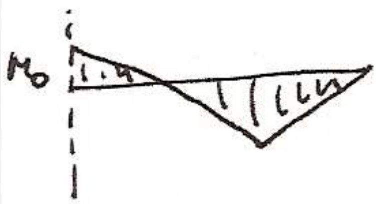
$\frac{h}{b} = 2$



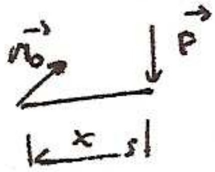


$\mu = 4 \quad \partial = 3 \quad s = 1$

$w_B = 0 = \frac{\partial W}{\partial F_B}$

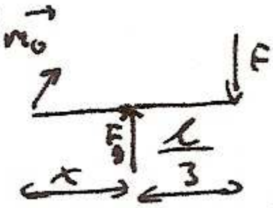


(I)



$M_o = -F \cdot x \qquad \frac{\partial M_o}{\partial F_B} = 0$

(II)



$M_o = -F\left(x + \frac{l}{3}\right) + F_B \cdot x$

$\frac{\partial M_o}{\partial F_B} = x$

$$0 = \frac{1}{E J_y}\left[\int_0^{\frac{l}{3}} -F \cdot x \cdot 0\, dx + \int_0^{l}\left[-F \cdot \left(x + \frac{l}{3}\right) + F_B \cdot x\right] \cdot x \cdot dx\right.$$

$$0 = \int_0^{l}\left[-F x^2 - F \cdot x \cdot \frac{l}{3} + F_B \cdot x^2\right] \cdot dx$$

$$0 = -F \cdot \frac{l^3}{3} - F \cdot \frac{l^3}{6} + F_B \cdot \frac{l^3}{3} \Rightarrow \frac{1}{2} F = \frac{1}{3} F_B \Rightarrow F_B = \frac{3}{2} F$$

(I) $M_o(0) = 0 \qquad M_o\left(\frac{l}{3}\right) = -F \cdot \frac{l}{3}$ (II) $M_o(0) = -F \cdot \frac{l}{3} \qquad M_o(l) = -\frac{4}{3} F \cdot l + \frac{3}{2} F \cdot l$

$= \frac{1}{6} F \cdot l = 0{,}17 F \cdot l$

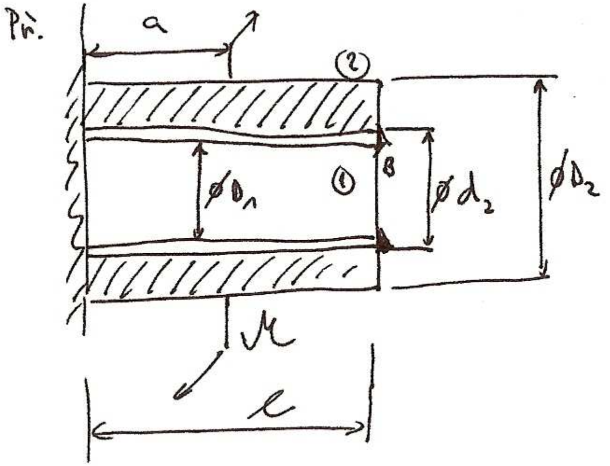
$M_{o\,MAX} = \frac{1}{3} F \cdot l$

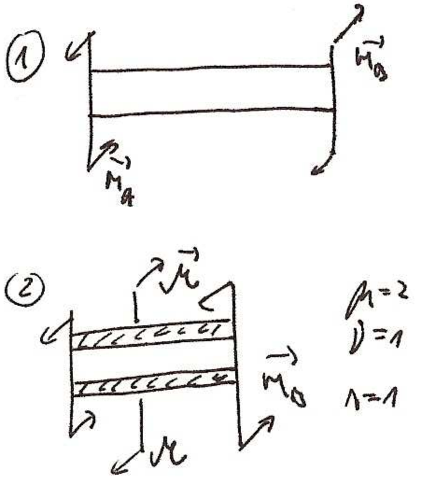
$$\frac{\sigma_K}{k_K} = \frac{6 \cdot F \cdot l \cdot 2}{3 B H^2} = \frac{4 F \cdot l}{H^3} \Rightarrow H$$

$$H = \sqrt[3]{\frac{4 \cdot F \cdot l \cdot k_K}{\sigma_K}} = 45\ mm$$

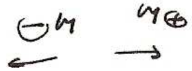
$B = 22{,}5\ mm$

Pr.



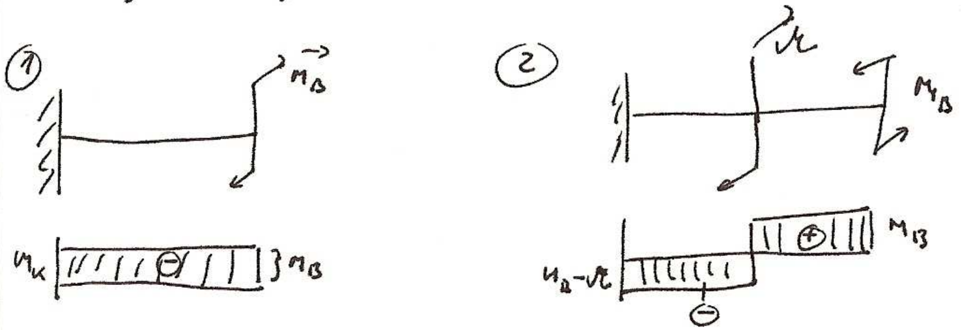


$\mu = 2$
$\nu = 1$
$n = 1$



$$\varphi_B^{(1)} = \varphi_B^{(2)}$$

$$\frac{\partial w_1}{\partial M_B} = -\frac{\partial w_2}{\partial M_B}$$



$$\frac{M_B \cdot l}{G \cdot J_p^{(1)}} = \frac{\mathcal{M} \cdot a}{G \cdot J_p^{(2)}} - \frac{M_B \cdot l}{G\, J_p^{(2)}}$$

$$M_B = \frac{a}{J_p^{(2)}\left(\frac{l}{J_p^{(1)}} + \frac{l}{J_p^{(2)}}\right)} \cdot \mathcal{M}$$

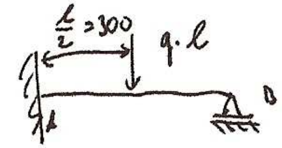
$$M_B = k\, \mathcal{M}$$

$$\frac{\tau_K}{2 k_K} = \frac{M_K}{W_K} \Rightarrow k_K = \frac{\tau_K \cdot W_K}{2 \cdot M_K}$$

(1) $k_{K1} = \frac{\tau_K \cdot W_K^{(1)}}{2 \cdot M_B}$ (2) $k_{K2} = \frac{\tau_K \cdot W_K^{(2)}}{2 \cdot M_B}$ (3) $k_{K3} = \frac{\tau_K \cdot W_K^{(2)}}{2 \cdot (M_B - \mathcal{M})}$

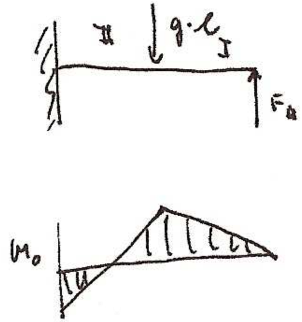
$k_K =$ [redacted] $\min(k_{Ki})$

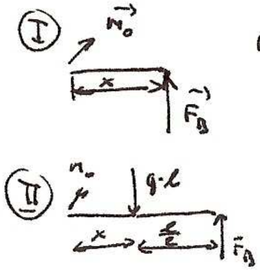
Pr.
$F_B = ?$



$n = 4 \quad \nu = 3 \quad s = 1$

$$w_B = 0 = \frac{\partial W}{\partial F_B}$$





$M_o = F_B \cdot x \qquad \frac{\partial M_o}{\partial F_B} = x$

$M_o = F_B \cdot \left(x + \frac{l}{2}\right) - q \cdot l \cdot x$

$$\frac{\partial M_o}{\partial F_B} = x + \frac{l}{2}$$

$$0 = \frac{1}{E J_y}\left[\int_0^{\frac{l}{2}} F_B \cdot x \cdot x \cdot dx + \int_0^{\frac{l}{2}} \left(F_B \cdot \left(x + \frac{l}{2}\right) - q \cdot l \cdot x\right) \cdot \left(x + \frac{l}{2}\right) dx\right]$$

$$0 = \int_0^{\frac{l}{2}} F_B \cdot x^2 \cdot dx + \int_0^{\frac{l}{2}} \left[F_B \cdot \left(x^2 + x \cdot l + \frac{l^2}{4}\right) - q \cdot l \cdot x^2\right] dx$$

$$0 = F_B \cdot \frac{l^3}{24} + F_B \cdot \frac{l^3}{24} + F_B \cdot \frac{l^3}{8} + F_B \cdot \frac{l^3}{8} - \frac{q \cdot l^4}{24} - \frac{q \cdot l^4}{16}$$

$$\frac{1}{3} F_B = \frac{5}{48} q \cdot l \Rightarrow \boxed{F_B = \frac{5}{16} q \cdot l}$$