VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ
FAKULTA STROJNÍHO INŽENÝRSTVÍ

PRUŽNOST A PEVNOST

Řešené příklady

Čtvrtek, 17. května 2007

Jan Tihlařík

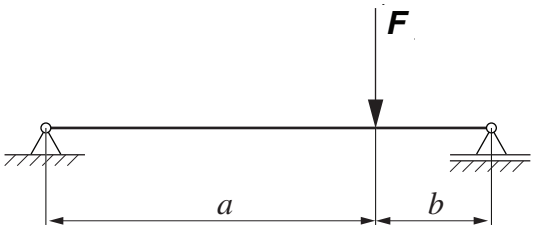
Řešené příklady v tomto dokumentu byly vyřešeny jako náhrada za písemnou práci v předmětu Pružnost a pevnost I. Příklady jsem řešil osobně a bez jakékoliv pomoci.

Jelikož nemám prostředky pro převedení psaného písma do digitální podoby (scanner, digitální fotoaparát), byl jsem nucen kreslit obrázky a matematické zápisy přímo v elektronické podobě (doufám že tuto námahu oceníte:).

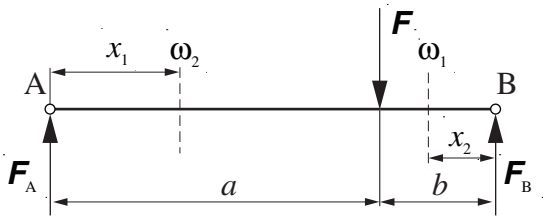
Za případné chyby, vzniklé nepozorností při spěchu a nervozitě v tomto časovém období se Vám předem omlouvám.

Jan Tihlařík

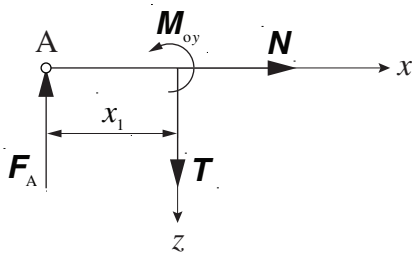
Přímý prut, zjištění VVÚ



Řešení



ω_2 :

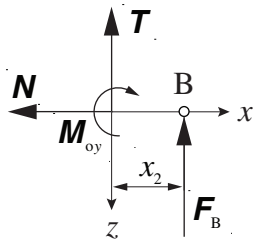


$$\sum F_x : N = 0$$

$$\sum F_z : T - F_A = 0 \Rightarrow T = F_A$$

$$\sum F_y : M_{oy} - F_A \cdot x_1 = 0 \Rightarrow M_{oy} = F_A \cdot x_1$$

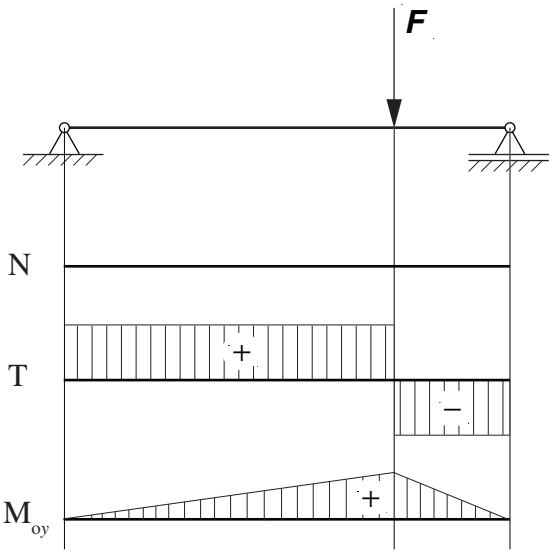
ω_1 :



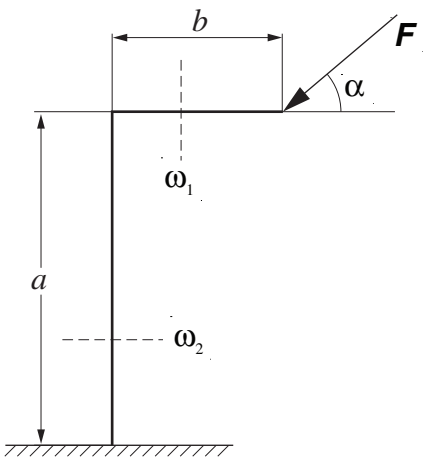
$$\sum F_x : N = 0$$

$$\sum F_z : -T - F_B = 0 \Rightarrow T = -F_B$$

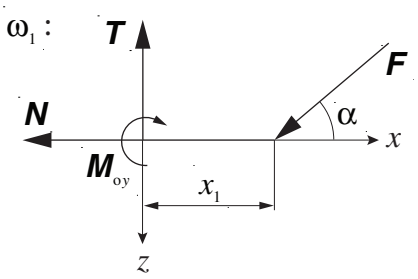
$$\sum F_y : -M_{oy} + F_B \cdot x_2 = 0 \Rightarrow M_{oy} = F_B \cdot x_2$$



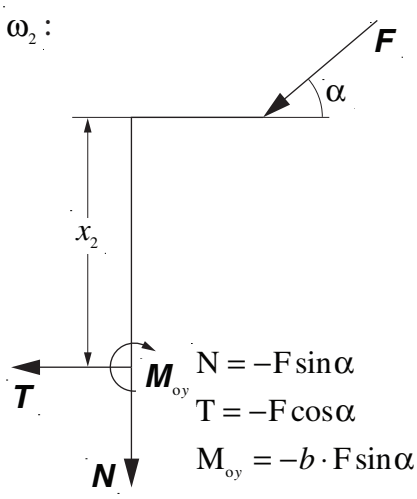
Zalomený prut, zjištění VVÚ



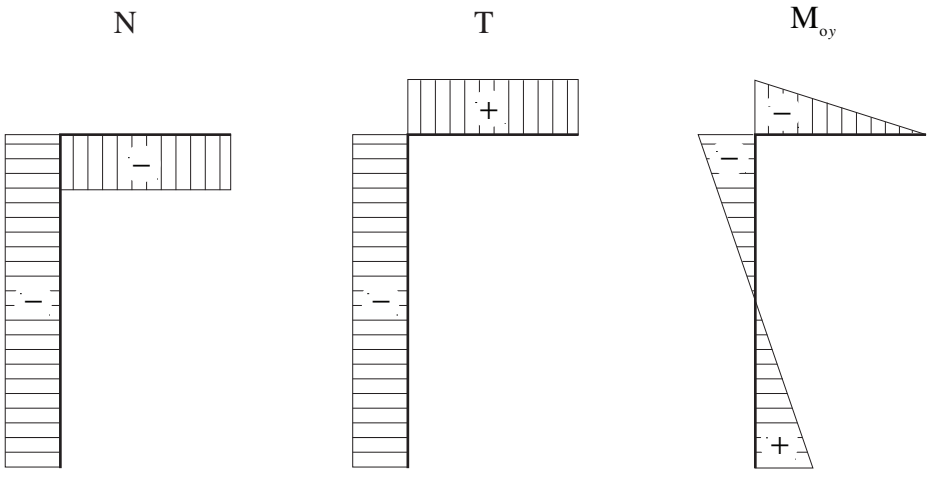
Řešení



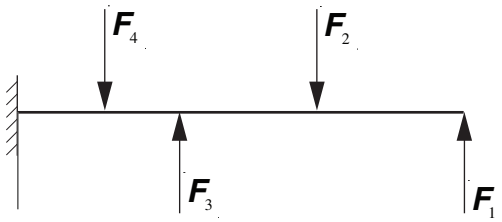
$$N = -F\cos\alpha$$
$$T = F\sin\alpha$$
$$M_{oy} = -F\sin\alpha \cdot x_1$$



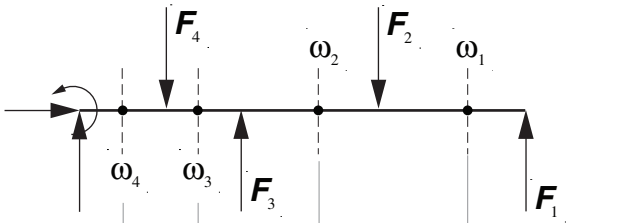
$$N = -F\sin\alpha$$
$$T = -F\cos\alpha$$
$$M_{oy} = -b \cdot F\sin\alpha + x_2 \cdot F\cos\alpha$$



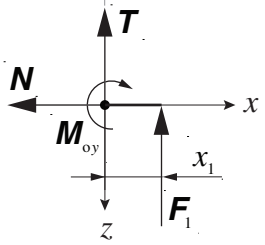
Přímý prut, zjištění VVÚ



Řešení

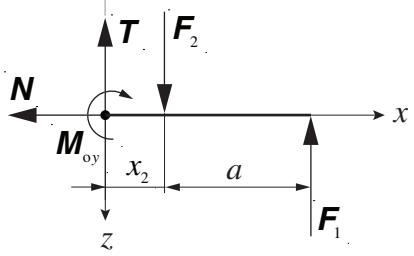


ω_1:



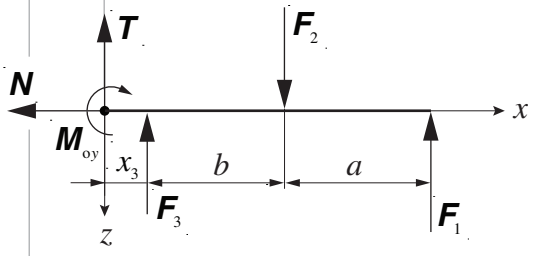
$$\sum \mathrm{F}_x : -\mathrm{N} = 0$$
$$\sum \mathrm{F}_y : -\mathrm{M}_{oy} + \mathrm{F}_1 \cdot x_1 = 0 \Rightarrow \mathrm{M}_{oy} = \mathrm{F}_1 \cdot x_1$$
$$\sum \mathrm{F}_z : -\mathrm{T} - \mathrm{F}_1 = 0 \Rightarrow \mathrm{T} = -\mathrm{F}_1$$

ω_2:



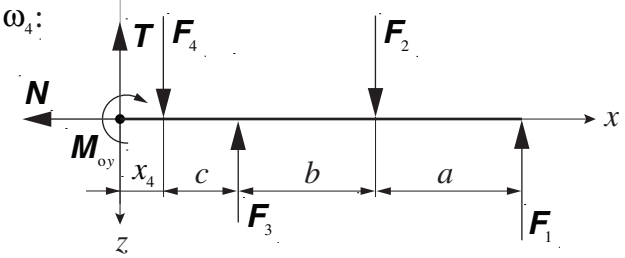
$$\sum \mathrm{F}_x : -\mathrm{N} = 0$$
$$\sum \mathrm{F}_y : -\mathrm{M}_{oy} + \mathrm{F}_1 \cdot (a + x_2) - \mathrm{F}_2 \cdot x_2 = 0$$
$$\Rightarrow \mathrm{M}_{oy} = \mathrm{F}_1 \cdot (a + x_2) - \mathrm{F}_2 \cdot x_2$$
$$\sum \mathrm{F}_z : \mathrm{F}_2 - \mathrm{F}_1 - \mathrm{T} = 0$$
$$\Rightarrow \mathrm{T} = \mathrm{F}_2 - \mathrm{F}_1$$

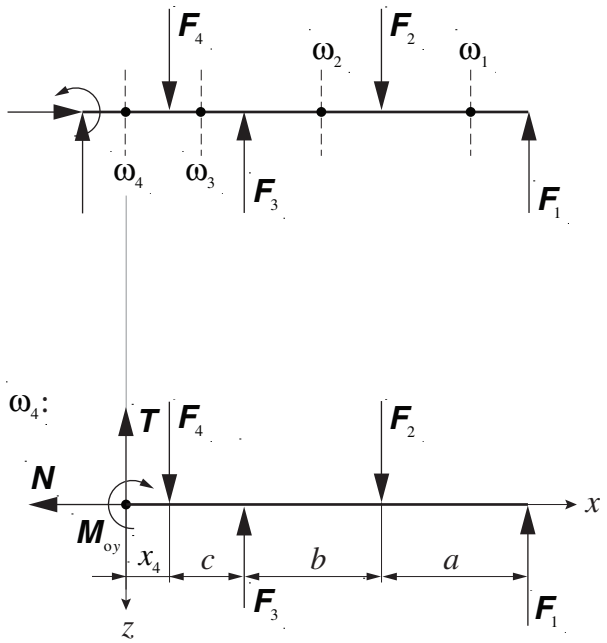
ω_3:



$$\sum \mathrm{F}_x : -\mathrm{N} = 0$$
$$\sum \mathrm{F}_y : -\mathrm{M}_{oy} + \mathrm{F}_1 \cdot (a + b + x_3) -$$
$$-\mathrm{F}_2 \cdot (b + x_3) + \mathrm{F}_3 \cdot x_3 = 0$$
$$\Rightarrow \mathrm{M}_{oy} = \mathrm{F}_1 \cdot (a + b + x_3) -$$
$$-\mathrm{F}_2 \cdot (b + x_3) + \mathrm{F}_3 \cdot x_3$$
$$\sum \mathrm{F}_z : \mathrm{F}_2 - \mathrm{F}_1 - \mathrm{F}_3 - \mathrm{T} = 0$$
$$\Rightarrow \mathrm{T} = \mathrm{F}_2 - \mathrm{F}_1 - \mathrm{F}_3$$

ω_4:





$$\sum \mathrm{F}_x : -\mathrm{N} = 0$$

$$\sum \mathrm{F}_y : -\mathrm{M}_{oy} + \mathrm{F}_1 \cdot (a+b+c+x_4) -$$

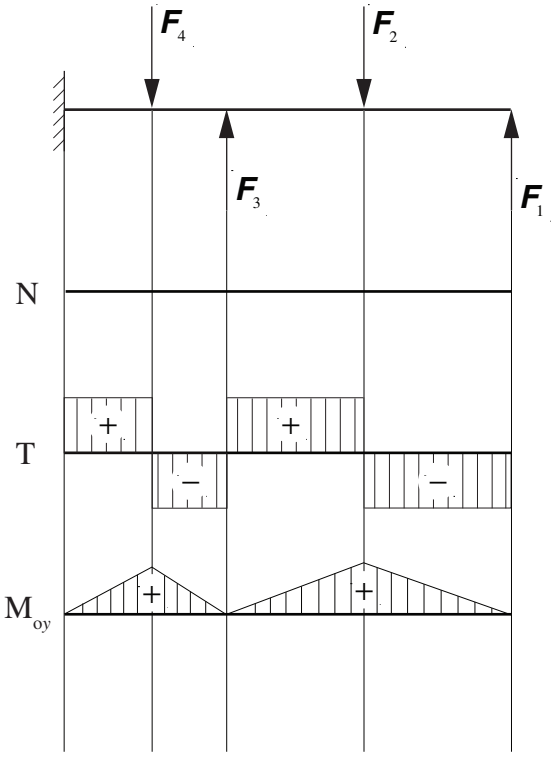
$$-\mathrm{F}_2 \cdot (b+c+x_4) + \mathrm{F}_3 \cdot (c+x_4) - \mathrm{F}_4 \cdot x_4 = 0$$

$$\Rightarrow \mathrm{M}_{oy} = \mathrm{F}_1 \cdot (a+b+c+x_4) -$$

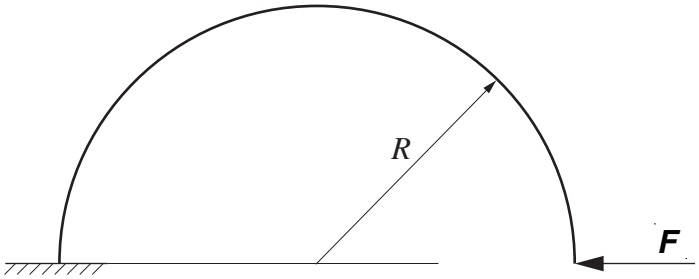
$$-\mathrm{F}_2 \cdot (b+c+x_4) + \mathrm{F}_3 \cdot (c+x_4) - \mathrm{F}_4 \cdot x_4$$

$$\sum \mathrm{F}_z : \mathrm{F}_4 + \mathrm{F}_2 - \mathrm{F}_1 - \mathrm{F}_3 - \mathrm{T} = 0$$

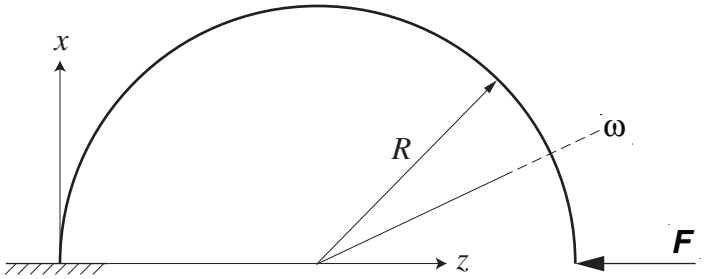
$$\Rightarrow \mathrm{T} = \mathrm{F}_4 + \mathrm{F}_2 - \mathrm{F}_1 - \mathrm{F}_3$$



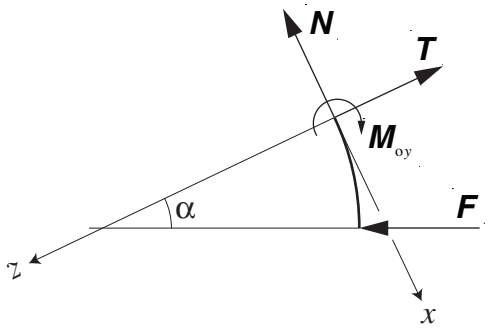
Vrub, výpočet napětí



Řešení

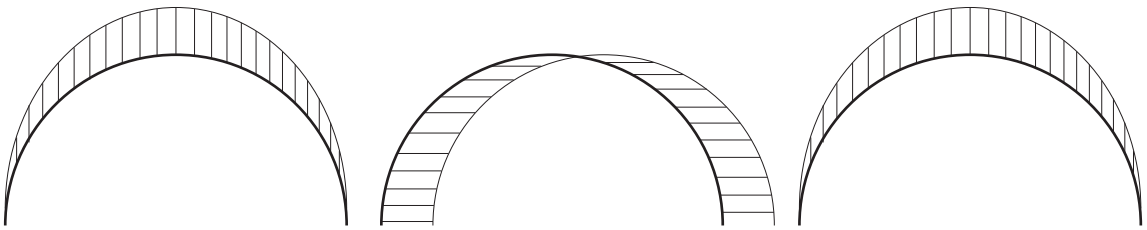


ω :

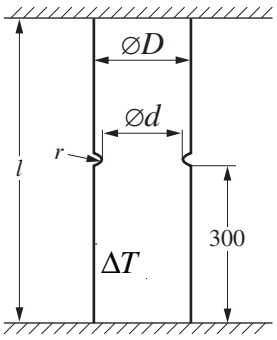


$$N = -F\sin\alpha$$
$$T = F\cos\alpha$$
$$M_{oy} = -F\sin\alpha \cdot R$$

N T M_{oy}

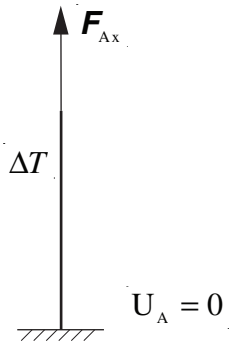


Vrub, výpočet napětí



$\varnothing D = 40$ mm
$\varnothing d = 36$ mm
$r = 2$ mm
$\Delta T = 120\,°\mathrm{C}$
$l = 2$ mm
$\alpha_T = 1{,}2 \cdot 10^{-5}\,\mathrm{K}^{-1}$

Řešení



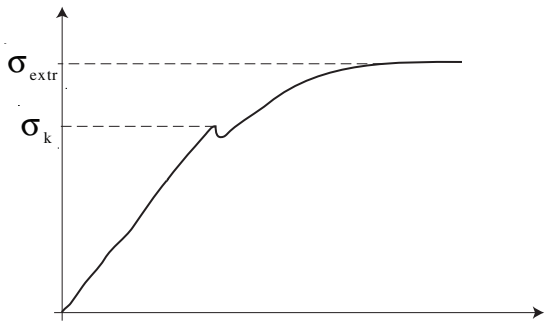
$$\frac{\partial \alpha}{\partial F_{Ax}} + \Delta T \alpha_T l = 0 \quad ; \quad \frac{N \cdot l}{E \cdot S} \cdot \frac{\partial N}{\partial F_{Ax}} + \Delta T \alpha_T l = 0$$

$$\frac{F_{Ax} l}{E \cdot S} = -\Delta T \alpha_T = 0$$

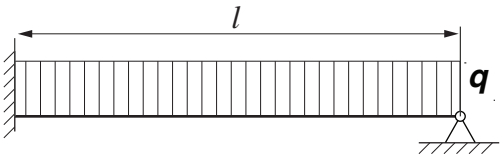
$$F_{Ax} = -\Delta T \cdot \alpha_T \cdot E \cdot S = -380\,\mathrm{kN}$$

Napětí u vrubu $\sigma_n = \dfrac{4F_{Ax}}{\pi d^2} = 373\ \mathrm{MPa}$ *(nominální napětí)*

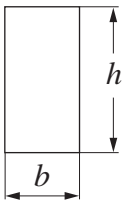
$$\sigma_{extr} = \alpha \cdot \sigma_n = 672\ \mathrm{MPa}$$
$$\sigma_k = \sigma_p \cdot 0{,}7 = 350\,\mathrm{MPa}$$



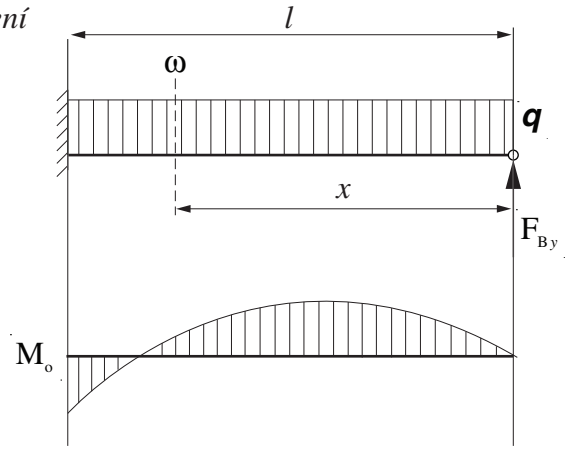
Přímý prut, výpočet bezpečnosti



$q = 20\ \text{kN}$
$l = 600\ \text{mm}$
$b = 20\ \text{mm}$
$h = 30\ \text{mm}$
$kk = ?$



Řešení



$w_B = 0 \qquad \omega_1: \quad M_o = F_{By} \cdot x - q \cdot \frac{x_2}{2}$

$F_{By} = ?$

$$w_B = \int_0^l \frac{M_o}{E \cdot J_y} \cdot \frac{\partial M_o}{\partial F_{By}} dx =$$

$$= \frac{1}{E \cdot J_y} \int_0^l \left(F_{By} \cdot x - q x^2 \right) x dx =$$

$$= \frac{1}{E \cdot J_y} \left(F_{By} \cdot \frac{l^3}{3} - q \cdot \frac{l^4}{8} \right) = 0$$

$$\Rightarrow F_{By} \cdot \frac{l^3}{3} - q \cdot \frac{l^4}{8} = 0 \Rightarrow F_{By} = q \cdot \frac{l^4}{8} \cdot \frac{3}{l^3}$$

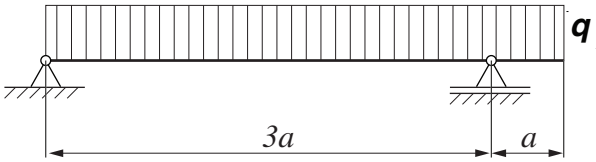
$$F_{By} = q \cdot l \cdot \frac{3}{8}$$

$$\frac{\partial M_o}{\partial x} = F_{By} - qx = 0 \Rightarrow x = \frac{F_{By}}{q} = \frac{3}{8} l$$

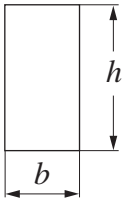
$$M_{o\,max} = \frac{3ql}{8} \cdot \frac{3l}{8} - q \frac{\left(\frac{3}{8} l \right)^2}{2} = \frac{9ql^2}{64} - \frac{9ql^2}{128} = 506\,\text{N} \cdot \text{m}$$

$$\frac{M_{o\,max}}{W_o} = \frac{\sigma_k}{kk} \Rightarrow \quad kk = \frac{\sigma_k \cdot bh^2}{6 M_{omax}} = 2{,}08$$

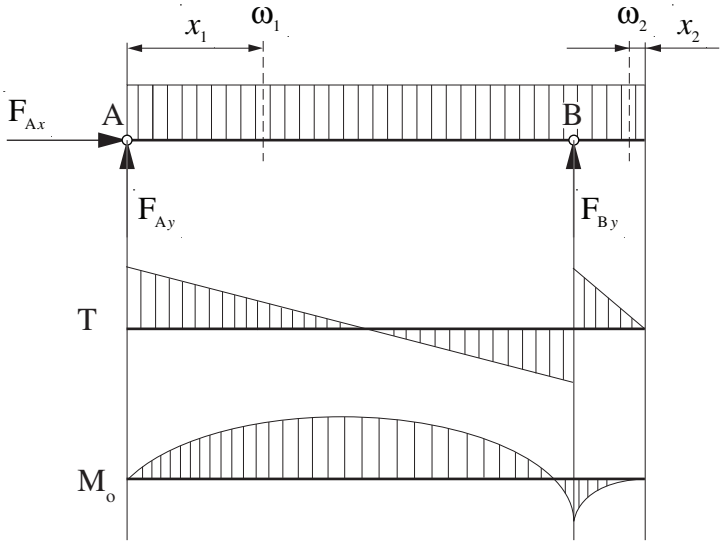
Přímý prut, ohyb



$a = 200$ mm
$q = 5\ \mathrm{kN \cdot m^{-1}}$
$h = 2b$
$kk = 2$
Materiál = 11420
$h, b = ?$



Řešení



$$\sum F_x = F_{Ax} = 0$$
$$\sum F_y = F_{Ay} + F_{By} - 4qa$$
$$\sum M_{Az} = F_{By} \cdot 3a - 8qa^2$$

$$F_{Ay} = -F_{By} + 4qa = \frac{4}{3}qa$$
$$F_{Ax} = 0$$
$$F_{By} = \frac{8qa^2}{3a} = \frac{8}{3}qa$$

ω_1: $T = F_{Ay} - q \cdot x_1$

$$M_{oI} = -F_{Ay} \cdot x_1 + \frac{qx_1^2}{2}$$

ω_2: $T = q \cdot x_2$

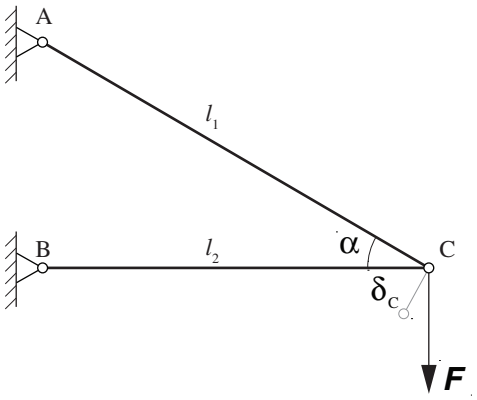
$$M_{oII} = -q\frac{x_2^2}{2} = -q\frac{a^2}{2} = -100\ \mathrm{N.m^{-1}}$$

$$\frac{\partial M_{oI}}{\partial x_1} = 0 \Rightarrow F_{Ay} = qx_1$$
$$\frac{4}{3}qa = -qx_1$$
$$\frac{4}{3}a = x_1 \qquad \Rightarrow \qquad M_{oI} = -\frac{16}{9}qa^2 + \frac{16}{18}qa^2 = 177$$

$$\sigma_k = \sigma_p \cdot 0,7 = 420 \cdot 0,7 = 294$$
$$\sigma_{max} = \frac{M_{o\,max}}{W_o} = \frac{\sigma_k}{kk} \Rightarrow \frac{6 \cdot M_{o\,max}}{bh^2} = \frac{\sigma_k}{kk}$$

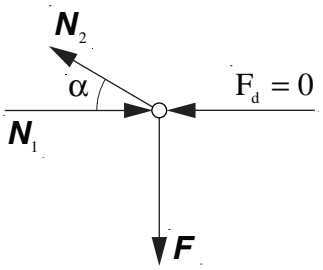
$$h = \sqrt[3]{\frac{12 \cdot M_{o\,max}}{\sigma_k}} = 2,44\ \mathrm{mm} \qquad b = \frac{h}{2} = \frac{2,44}{2} = 1,22\,\mathrm{mm}$$

Pruty, výpočet posuvu



$\alpha = 45°$
$l_1 = 14$ mm
$l_2 = 1000$ mm
$F = 10^5$ N
$kk = 2$

Řešení



$$\sum F_x : N_1 - F_d - N_2 \frac{\sqrt{2}}{2} = 0$$

$$\sum F_y : -F + N_2 \frac{\sqrt{2}}{2} = 0$$

$$N_2 = \sqrt{2} \cdot F$$

$$N_1 = F + F_d$$

$$\sigma = \frac{N_i}{S} = \frac{\sigma_k}{kk} \quad ; \quad \sigma_k = \sigma_p \cdot 0,75 = 370 \cdot 0,75 = 277,5$$

$$\frac{N_1}{S} = \frac{\sigma_k}{kk} = \frac{F + F_d}{a_1^2} \Rightarrow a_1 = \sqrt{\frac{(F + F_d) \cdot kk}{\sigma_k}} = \ldots$$

$$\frac{N_2}{S_2} = \frac{\sigma_k}{kk} = \frac{\sqrt{2} \cdot F}{a_2^2} \Rightarrow a_2 = \sqrt{\frac{\sqrt{2} \cdot F \cdot kk}{\sigma_k}} = \ldots$$

$$v_C = \frac{\partial w}{\partial F_d} = \sum_{i=1}^{2} \frac{N_i l_i}{E \cdot S_i} \cdot \frac{\partial N_i}{\partial F_d}$$

$$w_C = \frac{\partial w}{\partial F} = \sum_{i=1}^{2} \frac{N_i l_i}{E \cdot S_i} \cdot \frac{\partial N_i}{\partial F}$$

$$\delta_C = \sqrt{v_C^2 + w_C^2}$$